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SUSY SU(5)$\times S_4$ GUT flavor model for fermion masses and mixings with adjoint, large $\theta_{13}^{\mathrm{PMNS}}$

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Abstract: We propose an S_4 flavor model based on supersymmetric (SUSY) SU(5) GUT. The first and third generations of **10** dimensional representations in SU(5) are all assigned to be 1_1 of S_4. The second generation of **10** is to be 1_2 of S_4. Right-handed neutrinos of singlet **1** and three generations of $\bar{\mathbf{5}}$ are all assigned to be 3_1 of S_4. The VEVs of two sets of flavon fields are allowed a moderate hierarchy, that is $\langle \Phi^\nu \rangle \sim \lambda_c \langle \Phi^e \rangle$. Tri-Bimaximal (TBM) mixing can be produced at both leading order (LO) and next to next to leading order (NNLO) in neutrino sector. All the masses of up-type quarks are obtained at LO. We also get the bottom-tau unification $m_\tau = m_b$ and the popular Georgi-Jarlskog relation $m_\mu = 3m_s$ as well as a new mass relation $m_e = \frac{8}{27} m_d$ in which the novel Clebsch-Gordan (CG) factor arises from the adjoint field H_{24}. The GUT relation leads to a sizable mixing angle $\theta_{12}^e \sim \theta_c$ and the correct quark mixing matrix V_{CKM} can also be realised in the model. The resulting CKM-like mixing matrix of charged leptons modifies the vanishing θ_{13}^ν in TBM mixing to a large $\theta_{13}^{\mathrm{PMNS}} \simeq \theta_c/\sqrt{2}$, in excellent agreement with experimental results. A Dirac CP violation phase $\phi_{12} \simeq \pm\pi/2$ is required to make the deviation from θ_{12}^ν small. We also present some phenomenological numerical results predicted by the model.

Keywords: Phenomenological Models, Supersymmetry Phenomenology

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Contents

1 Introduction

In the Standard Model (SM) of particle physics the charged fermions, quarks and charged leptons, are massive fermions, while neutrinos are massless in SM. However the deficit of the observed neutrinos with respect to the theoretical predicted ones leads to the two famous puzzles in neutrino physics, i.e., the longstanding solar neutrino puzzle [1–3] and the atmospherical neutrino anomaly [4–12] before 1998. The puzzles can be explained through the neutrino oscillation mechanism, which indicate that the neutrinos are also massive and lepton flavors are mixed. The discovery of neutrino oscillations [13, 14] convinced people that neutrinos have tiny masses. Meanwhile the seesaw mechanism [15–18] seems to be a graceful solution to answer why neutrino masses are small. Nevertheless, the flavor mixing pattern

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with the observed mixing angles can not be explained through seesaw mechanism. Solar and atmospherical neutrino oscillation experiments have measured leptonic mixing angles with great accuracy. The resulting lepton mixing Pontecorvo-Maki-Nakagawa-Sakata matrix $U_{\rm PMNS}$ [19, 20] can be well compatible with the simple Tri-Bimaximal (TBM) mixing pattern, introduced by Harrison, Perkins and Scott [21, 22]:

$$U_{\rm TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{1.1}$$

which predicts that $\theta_{23}^{\nu} = \frac{\pi}{4}$ and $\theta_{12}^{\nu} = \arcsin\frac{1}{\sqrt{3}}$, but $\theta_{13}^{\nu} = 0$. The two nonzero leptonic mixing angles θ_{12}^{ν} and θ_{23}^{ν} are predicted to be rather large by contrast with the quark mixing angles, which are known to be very small [23]. Besides the Tri-Bimaximal mixing pattern *ansatz*, similar simple mixing patterns with vanishing θ_{13}^{ν} were proposed, such as Bimaximal (BM) [24–26], Golden-Ratio (GR) [27–31] and Democratic [32–34] mixing patterns. The simple patterns suggest some kind of underlying non-Abelian discrete flavor symmetry G_f would exist in the lepton sector at least. Indeed the mixing patterns have been paid a lot of attention in the flavor model buidlding community. For the flavor models based on the typical discrete symmetries, please see refs. [35–39] for a review. The models based on continuous groups have been proposed [40–47]. By adding higher order corrections, most of the models can give rise to a non-zero reactor angle $\theta_{13} \sim \mathcal{O}(\lambda_c^2)$, with $\lambda_c = \sin\theta_c \simeq 0.22$ being Wolfenstein parameter [48], where θ_c is the Cabibbo angle. The resulting small θ_{13} was within the range of global fits [49] before the determinate large θ_{13} Daya Bay [50, 51] neutrino experiment measured. The deviations from the TBM values of θ_{12} and θ_{23} are also at most $\mathcal{O}(\lambda_c^2)$ when subleading effects are included, which is in agreement at 3σ error range with the experimental data or say global fits.

The Daya Bay Collaboration [50, 51] now has confirmed a larger θ_{13} with a significance of 7.7 (the first is 5.2) standard deviations from the reactor $\overline{\nu}_e \to \overline{\nu}_e$ oscillations. The best-fit result in 1σ range is

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010(\text{stat}) \pm 0.005(\text{syst}), \tag{1.2}$$

which is equivalent to $\theta_{13} \simeq 8.7^\circ \pm 0.8^\circ$. And the RENO [52] also reported that

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}). \tag{1.3}$$

The updated Daya Bay [53] data is measured to remarkable accuracy: $\sin^2 2\theta_{13} = 0.084 \pm 0.005$ or $\theta_{13} \approx 8.4^\circ \pm 0.2^\circ$. Even before the Daya Bay result, however, there have emerged direct evidence of large θ_{13} from T2K [54], MINOS [55] and Double Chooz [56]. The accurate nonzero reactor angle θ_{13} implies Tri-Bimaximal mixing pattern and else would be ruled out. However there still exists the possibility of maintaining the mixing angles $\theta_{12}^{\nu} = \arcsin\frac{1}{\sqrt{3}}$ and $\theta_{23}^{\nu} = \frac{\pi}{4}$ which TBM predicted as the leading order (LO) result of a model. The phenomenological favored value $\theta_{13} \approx \lambda_c/\sqrt{2}$, although now is intension with the updated data, can be derived from some proper corrections. In a general scheme consideration for obtaining the large θ_{13} from zero, the most popular and well motivated correction to TBM mixing is the contributions from charged lepton mixing, especially inducing a sizable $\theta_{12}^e \sim \theta_c$ is viable in the Grand Unified Theories (GUT) flavor models, see [57, 58] as example. The gauge symmetry groups are usually chose as SU(5), SO(10) or Pati-Salam context SU(4)$_C\times$ SU(2)$_L\times$ SU(2)$_R$. The simplest GUT gauge symmetry group is SU(5) [59], in which matter fields of standard model are assigned to be $\bar{\mathbf{5}}$ and $\mathbf{10}$ dimensional representations. In a generical GUT scheme the down-type quark and charged lepton Yukawa matrices are the crucial factor to produce GUT relations that connect θ_{13} with Cabibbo angle, see [60, 61]. We remark that a series of models based on discrete flavor symmetry group together with a GUT gauge group have been proposed, for example the SU(5) $\times A_4$ [62–64], SU(5)$\times S_4$ [26, 65–69] and SU(5)$\times T'$ [70], SO(10)$\times A_4$ [71–73], SO(10)$\times S_4$ [74–80], SO(10)$\times PSL_2(7)$ [81, 82] and SO(10)$\times\Delta_{27}$ [83]. Most of the GUT flavor models also generically give rise to $\theta_{13} \sim \mathcal{O}(\lambda_c^2)$, while only few models based on different ansatz, such as Bimaximal [25, 26], or empirical relation such as Quark-Lepton Complementarity (QLC) [78] may lead to sizable $\theta_{13} \sim \mathcal{O}(\lambda_c)$. The other way to achieve the sizable θ_{13} is the introduction of non-singlets such as the SU(5) adjoint fields $\mathbf{24}$, which split the heavy messenger masses and give rise to new novel GUT Yukawa coupling ratios of quark-lepton. For the realistic GUT flavor models based on the mechanism one can refer to, such as [84–86].

In this paper we propose a SUSY SU(5) GUT flavor model, with $S_4\times Z_4\times Z_6\times Z_5\times Z_2$ as flavor symmetry groups. The flavor symmetry S_4 can be spontaneously broken by Vacuum Expectation Values (VEV) of flavon fields in Φ which is divided into Φ^e in charged fermion sector and Φ^ν in neutrino sector. The assumption we adopted is similar with ref. [87], of which the VEVs of Φ^e and Φ^ν allow a moderate hierarchy: $\langle\Phi^\nu\rangle \sim \lambda_c\langle\Phi^e\rangle$. The dynamical tricky assumption makes the $\theta_{13}^{\mathrm{PMNS}}$ around $\mathcal{O}(\lambda_c)$ possible. The neutrino masses are simply generated through type-I see-saw mechanism. Tri-Bimaximal mixing pattern is produced exactly at LO, and even still holds exactly at next to next to leading order (NNLO) in neutrino sector, which is a salient feature of our model. The charged fermion mass hierarchies are controlled by spontaneously broken of the flavor symmetry without introducing Froggatt-Nielsen mechanism [88]. Both up- and down-type quarks obtain their masses with proper order of magnitude and the correct quark mixing matrix V_{CKM} can be realised in the model.

The masses of charged leptons are similar with those of down-type quarks regardless of the different group-theoretical Clebsch-Gordan (CG) coefficients. Due to the introduction of an adjoint field H_{24}, the resulting novel CG factors lead to a new mass ratio between electron and down quark, namely $m_e/m_d = 8/27$, which is a phenomenological favored result. The famous Georgi-Jarlskog relation $m_\mu = 3m_s$ and bottom-tau unification $m_\tau = m_b$ are also maintained in the model. The mixing angle $\theta_{12}^e \sim \theta_c$ is also achieved by the specific GUT-scale relation between the angle and mass ratio m_e/m_μ [57]. Finally the CKM-like mixing matrix of charged leptons would modify the vanishing θ_{13}^ν in TBM mixing to a large $\theta_{13}^{\mathrm{PMNS}} \simeq \theta_c/\sqrt{2}$, in excellent agreement with experimental determinations.

The paper is organized as follows. In section 2 we discuss the basic strategic considerations for obtaining the large $\theta_{12}^e \sim \lambda_c$, including the desired Yukawa textures, the hierarchy assumption on flavon VEVs and the role of the adjoint H_{24}. In section 3 we introduce all matter fields and flavons, and the predictions for the fermions masses and mixings at LO are presented. In section 4 the vacuum alignment are justified by minimizing the potential. Section 5 is devoted to the subleading corrections to the VEVs of the flavons, the LO masses and mixings of fermions. In section 6 we show a bit of phenomenology of the model predicted in numerical results. Section 7 is our conclusion.

2 The strategy and assumptions

In a large class of flavor models that give arise to TBM and else mixing patterns with or without GUT context, the angle θ_{13} is usually about $\mathcal{O}(\lambda_c^2) \sim 3^\circ$ by taking subleading corrections into account. In order to obtain the sizable mixing angle $\theta_{13} \sim \mathcal{O}(\lambda_c)$, the most popular and well-motivated correctional approach can be provided by the large charged lepton mixing contributions. The fully lepton mixing matrix $U_{\text{PMNS}} = V_L^{\ell\dagger} U_\nu$ in which U_ν is usually taken TBM, BM or GR as first order approximation, while V_L^ℓ is not uniquely determined. The general model-independent studies on the deviations from TBM mixing with the contributions of V_L^ℓ have been proposed, see refs. [89–94] as examples.

One of the aims in the work is to generate a large mixing θ_{12}^e in charged lepton sector. In the context of unified theory the Yukawa matrices of charged leptons and down quarks are unified in single joint operators, which provide a possible approach to generating a larger mixing angle $\theta_{12}^e \simeq \lambda_c$. Nevertheless, the resulting mixing angle and masses should satisfy some specific GUT-scale relations in order to fit the realistic phenomenological constrains. However the traditional unified operators which give rise to some GUT-scale mass relations, such as the popular Georgi-Jarlskog (GJ) relations [95] $m_\mu = 3m_s$ and $m_e \simeq \frac{1}{3} m_d$ will lead to $\theta_{12}^e \simeq \lambda_c/3$. As consequence the GJ factor of 3 leads to $\theta_{13} \simeq \lambda_c/3\sqrt{2}$ in a large class of GUT flavor models, which now contradicts with experimental data. For achieving $\theta_{13} \simeq \lambda_c/\sqrt{2}$ in GUT flavor models, the basic strategic considerations has been suggested in [57, 60, 61, 96]. A different strategy without GUT can be seen in [97, 98]. Note that the Yukawa matrices we mentioned are taken to be equivalent to the mass matrices, or say $m_{ij} = y_{ij} v$ with the value of Higgs VEV v being fixed at the electroweak scale Λ_{EW}. Without loss of generality we consider the upper 2×2 part of down quarks Yukawa matrices with a vanishing 11-entry for simplicity, the desired Yukawa matrices that lead to $\theta_{12}^{d,e} \simeq \lambda_c$ are generically of the form

$$Y_D \propto \begin{pmatrix} 0 & a \\ b & c \end{pmatrix} \Longrightarrow \quad Y_e \propto \begin{pmatrix} 0 & c_a a \\ c_b b & c_c c \end{pmatrix}^T = \begin{pmatrix} 0 & c_b b \\ c_a a & c_c c \end{pmatrix} \tag{2.1}$$

in which parameters a, b and c are suitable complex numbers that can give rational magnitude order of the quark masses $m_d \simeq |ab/c|$ and $m_s \simeq |c|$ and the desired mixing angle $\theta_{12}^d \simeq |b/c| \simeq \lambda_c$, thus the Cabibbo angle can be derived. Similarly the charged lepton masses $m_e = |\frac{c_a a c_b b}{c_c c}|$ and $m_\mu \simeq c_c c$ and the mixing angle $\theta_{12}^e \simeq |\frac{c_a a}{c_c c}|$, where the CG factors $c_{a,b,c}$ are uniquely determined by SU(5) contractions. The mixing angle θ_{12}^e is, however, constrained by the GUT-scale relation between mixing angle and the mass ratio [57]:

$\theta_{12}^e = |\frac{c_c}{c_b}|\frac{m_e}{m_\mu}\frac{1}{\theta_{12}^d}$. The mixing angle $\theta_{12}^e \simeq \lambda_c$ can be achieved only when the CG ratio $|\frac{c_c}{c_b}| \sim \mathcal{O}(2/\lambda_c)$ is satisfied since the order of the mass ratio $\frac{m_e}{m_\mu} \sim \lambda_c^3/2$ and $\theta_{12}^d \sim \lambda_c$.

For satisfying the ratio the CG factors in the Yukawa matrix Y_e should be chose properly such as the novel ones in [57, 99]. To be specific the structures of Y_e are given as following

$$Y_e \propto \begin{pmatrix} 0 & -\frac{1}{2}b \\ 6a & 6c \end{pmatrix} \qquad \text{or} \qquad \begin{pmatrix} 0 & \frac{4}{9}b \\ \frac{9}{2}a & \frac{9}{2}c \end{pmatrix}, \tag{2.2}$$

which predict $|\frac{c_c}{c_b}| = 12$ or $|\frac{c_c}{c_b}| = 81/8$. Both Yukawa textures satisfy the ratio $|\frac{c_c}{c_b}| \sim \mathcal{O}(2/\lambda_c)$ and also require $a/c \simeq \lambda_c$. The above structures demand the introduction of non-singlet fields, such as adjoint fields **24** in either the numerator (first Y_e) or the denominator (second Y_e) of the effective operators. Only in the methods can we obtain new predictions of Yukawa coupling ratios. For such realistic models one can refer to [84–86]. However we find that there exists another possible structure of Y_e which preserves the Georgi-Jarlskog relation and the ratio $|\frac{c_c}{c_b}| = 81/8$ still holds. The texture of Y_e is the metamorphosis of the second Y_e in eq. (2.2)

$$Y_e \propto \begin{pmatrix} 0 & -\frac{8}{27}b \\ -3a & -3c \end{pmatrix}. \tag{2.3}$$

It is an important goal to obtain above Y_e in the model. The CG factor -3 arises from the conventional GJ Higgs $H_{\overline{45}}$, and the novel CG factor $-\frac{8}{27}$ is due to the VEV of the adjoint H_{24} appears in the denominator of effective operators. The non-singlet fields are essential to make the masses (size around GUT scale) of the components of the messenger split by CG coefficients [99]. Finally the CG factors enter inversely in the desired Yukawa matrix elements and the new predictions arise.

Another difficulty in generating the desired Yukawa structures and hence the mixings is the vacuum alignments which arise from the spontaneously broken flavor symmetry. Denoting the general scalar flavon fields with Φ, S_4 can be spontaneously broken by the VEVs of flavons in Φ which is divided into two sets: $\Phi^e = \{\varphi, \eta, \chi, \sigma, \vartheta, \xi, \rho\}$ in charged fermion sector and $\Phi^\nu = \{\phi, \Delta, \zeta\}$ in neutrino sector. We also assume a moderate hierarchy between the VEVs of Φ^e and Φ^ν: $\langle\Phi^\nu\rangle \sim \lambda_c\langle\Phi^e\rangle$, to induce $a/c \simeq \lambda_c$. For sake of convenience two small expansion parameters ϵ and δ are introduced as

$$\frac{\langle\Phi^e\rangle}{\Lambda} \sim \epsilon \sim \lambda_c^2, \quad \frac{\langle\Phi^\nu\rangle}{\Lambda} \sim \delta \sim \lambda_c^3. \tag{2.4}$$

The assumption provides a possibility to generate the desired relation between (21) and (22) elements in eq. (2.2) and eq. (2.3). In fact the angle $\theta_{12}^e \sim \lambda_c$ in the model is produced by the ratio $\langle\Phi^\nu\rangle/\langle\Phi^e\rangle \sim \lambda_c$. The specific dynamical tricks are in principle allowed by "separated" scalar potential, which is guaranteed by the auxiliary Abelian flavor symmetry $\mathcal{G}_A$. The $\mathcal{G}_A$ separates the scalar potential of the flavons in Φ^e and Φ^ν generically as follows

$$V(\Phi^e, \Phi^\nu) = V_\nu(\Phi^\nu)|_{\text{LO}} + V_e(\Phi^e)|_{\text{LO}} + V(\Phi^\nu, \Phi^e)|_{\text{sub}} \tag{2.5}$$

where $V(\Phi^\nu, \Phi^e)|_{\text{sub}}$ is the subleading scalar potential, at least at NLO, and is usually called "partially" separated. At LO the two sectors are naturally separated, but it is not the case for subleading corrections at NLO and/or NNLO. The scalar potential is "fully" separated while $V(\Phi^\nu, \Phi^e)|_{\rm sub} = V(\Phi^e)|_{\rm sub}$ or $V(\Phi^\nu, \Phi^e)|_{\rm sub} = V(\Phi^\nu)|_{\rm sub}$, which makes a hierarchy between $\langle\Phi^e\rangle$ and $\langle\Phi^\nu\rangle$ possible, see Lin's work in ref. [87]. In our model the auxiliary flavor group is chosen as $\mathcal{G}_A = Z_4 \times Z_6 \times Z_5 \times Z_2$.

However in the present GUT flavor model the "fully" separated scalar potential is not exactly the same as that in Lin's proposal. The subleading scalar potential is separated as

$$V(\Phi^\nu, \Phi^e)|_{\rm sub} = V(\Phi^\nu)|_{\rm NNLO} + V(\Phi^e, \Phi^\nu)|_{\rm NLO} + \cdots \tag{2.6}$$

where the dots stand for higher order subleading terms. The scalar potential of Φ^ν is separated at both LO and NNLO, thus the magnitude of $\langle\Phi^\nu\rangle/\Lambda$ is not necessarily the same as that of $\langle\Phi^e\rangle/\Lambda$. Actually it is feasible to build a model for TBM based on the S_4 symmetry with the allowed hierarchy in eq. (2.4). The hierarchy assumption not only gives arise to the large $\theta^e_{12} \sim \lambda_c$, but also produces the correct up quark mass at LO. The mass hierarchies of up-type quarks are roughly as $m_u : m_c : m_t = \lambda_c^8 : \lambda_c^4 : 1$, we note that λ_c^8 can be not only given by ϵ^4, but also by $\epsilon\delta^2$. Indeed the mass of up quark is given by the flavon combinations of order $\epsilon\delta^2$ at leading order operators. To a certain extent the hierarchical VEVs of Φ^e and Φ^ν are even necessary in the present model.

3 The construction of the model

In this section we introduce the SUSY SU(5)$\times S_4$ GUT flavor model with the auxiliary Abelian $Z_4 \times Z_6 \times Z_5 \times Z_2$ shaping symmetries. The flavor symmetry group S_4 is the permutation group of four objects, as well as the invariance group of octahedron and cube. It has 24 elements, which can be generated by two basic permutations S and T as the generators:

$$S^4 = T^3 = (ST^2)^2 = 1 \tag{3.1}$$

In group theory the generic permutation can be expressed by $(1, 2, 3, 4) \to (n_1, n_2, n_3, n_4) \equiv (n_1 n_2 n_3 n_4)$. The two basic permutations $S = (2341)$ and $T = (2314)$ is used to generate the elements of S_4. The group has five inequivalent irreducible representations: two three-dimensional representations 3_1 and 3_2, one 2-dimensional 2, and two one-dimensional 1_1 and 1_2 representations. The multiplication rules are presented as follows:

$$\begin{gathered}
1_1 \otimes r = r \otimes 1_1 = r, \quad 1_2 \otimes 1_2 = 1_1, \quad 1_2 \otimes 2 = 2, \quad 1_2 \otimes 3_1 = 3_2, \quad 1_2 \otimes 3_2 = 3_1, \\
2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2, \quad 2 \otimes 3_1 = 3_1 \oplus 3_2, \quad 2 \otimes 3_2 = 3_1 \oplus 3_2, \\
3_1 \otimes 3_1 = 3_2 \otimes 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2, \quad 3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_1 \oplus 3_2
\end{gathered} \tag{3.2}$$

The detailed irreducible representation matrices and the matrices of generators S, T are presented in appendix A. In the model matter fields, Higgs and flavon fields are assigned to be different representations of gauge group SU(5). All matter fields in SU(5) are unified into $\bar{\mathbf{5}}$ and $\mathbf{10}$ dimensional representations, denoted by F and $T_{1,2,3}$, respectively. The Higgs fields include the SU(5) $\mathbf{5}$, $\bar{\mathbf{5}}$, $\mathbf{45}$ and $\overline{\mathbf{45}}$-dimensional representations. The only one adjoint $\mathbf{24}$-dimensional field, H_{24}, is the key to obtain the desired CG factors. The

Field	T_3	T_2	T_1	F	N^c	H_5	$H_{\bar{5}}$	H_{45}	$H_{\overline{45}}$	H_{24}
SU(5)	**10**	**10**	**10**	$\bar{\mathbf{5}}$	**1**	**5**	$\bar{\mathbf{5}}$	**45**	$\overline{\mathbf{45}}$	**24**
S_4	1_1	1_2	1_1	3_1	3_1	1_1	1_1	1_2	1_1	1_1
Z_4	1	1	-1	1	1	1	-1	i	i	-1
Z_6	ω	$-\omega$	1	ω^2	1	$-\omega$	ω	$-\omega$	$-\omega^2$	ω
Z_5	1	ω	ω^2	1	1	1	ω^4	1	ω^3	1
Z_2	1	1	-1	1	-1	1	-1	-1	-1	1
$\mathrm{U}(1)_R$	1	1	1	1	1	0	0	0	0	0

Table 1. Transformation properties of the matter fields and Higgs fields in the model, where $\omega = e^{i\pi/3}$ for Z_6 group and $\omega = e^{i2\pi/5}$ for Z_5 group.

right handed neutrinos N^c and all flavon fields in Φ are SU(5) gauge singlets **1**. All the fields are also assigned to be different representations of the flavor symmetry group S_4 and the auxiliary shaping symmetries Z_N. The first and third generations of **10** dimensional representations T_1 and T_3 are all assigned to be 1_1 of S_4 singlet. The second generation of **10** dimensional representation T_2 is assigned to be 1_2 of S_4. Right-handed neutrinos N^c and three generations of $\bar{\mathbf{5}}$ F are all assigned to be 3_1 of S_4. The Higgs $H_{5,\bar{5},\overline{45}}$ and the adjoint H_{24} are all assigned to be 1_1 of S_4, while H_{45} is to be 1_2 of S_4. The flavon fields in Φ, which include all possible S_4 representations, are introduced to break the S_4 flavor symmetry spontaneously. To be specific, the left-handed down-type quarks in three colors and doublet leptons are collected in SU(5) representation $\bar{\mathbf{5}}$ as

$$F = (d^c_R \quad d^c_B \quad d^c_G \quad e \quad -\nu) \tag{3.3}$$

whereas the representation **10** contains $\mathrm{SU}(2)_L$ doublet quarks as well as up-type quark singlet and charged leptons

$$T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u^c_G & u^c_B & -u_R & -d_R \\ u^c_G & 0 & -u^c_R & -u_B & -d_B \\ -u^c_B & u^c_R & 0 & -u_G & -d_G \\ u_R & u_B & u_G & 0 & -e^c \\ d_R & d_B & d_G & e^c & 0 \end{pmatrix}_i \tag{3.4}$$

where $i = 1, 2, 3$ indicates the fermions family indices of standard model (SM), and R, B, G stand for the color indices. The matter fields and Higgs in the $\mathrm{SU}(5)\times S_4 \times Z_4 \times Z_6 \times Z_5 \times Z_2$ model, with their transformation properties under the flavor symmetry group, are listed in table 1. The flavon fields and the additional gauge singlets Φ_0, the driving fields φ_0, η_0, χ_0, σ_0, ξ_0 and ϕ_0, Δ_0, ζ_0, are listed in table 2. We also introduce a global $\mathrm{U}(1)_R$ continuous symmetry which meant a R-parity discrete subgroup. The driving fields carry +2 $\mathrm{U}(1)_R$ charge, which made them linearly appear in the superpotential. Matter fields and heavy right-handed neutrinos are charged with +1 $\mathrm{U}(1)_R$ charge, while all Higgs fields and flavons are uncharged.

In the following we shall present the model in detail. The masses and mixings of fermions arise from the spontaneously flavor symmetry breaking by the flavon fields acquiring the VEVs. The alignment directions of the vacua are crucial to generate the observed

Field	φ	η	χ	σ	ϑ	ξ	ρ	ϕ	Δ	ζ	φ_0	η_0	χ_0	σ_0	ξ_0	ϕ_0	Δ_0	ζ_0
S_4	3_1	2	3_2	1_1	1_1	3_1	1_1	3_1	2	1_2	3_1	1_1	3_2	1_1	3_1	3_2	2	1_1
Z_4	-1	-1	1	$-i$	i	i	i	1	1	-1	1	1	1	-1	-1	1	1	1
Z_6	ω^2	ω^2	$-\omega$	-1	-1	ω	ω	1	1	1	ω^2	ω^2	ω^2	1	$-\omega$	1	1	1
Z_5	ω	ω	ω^2	ω^4	ω^4	ω	ω	1	1	1	ω^3	ω^3	ω^3	ω^2	ω^3	1	1	1
Z_2	-1	-1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
$\mathrm{U}(1)_R$	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2

Table 2. Transformation properties of flavons and driving fields in the model.

mass hierarchies and mixings. For the time being the VEVs of the scalar components of the flavon fields are justified as the natural solutions of the scalar potential in section 4, which indicate the structures as following

$$\langle\varphi\rangle = v_\varphi \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \langle\eta\rangle = v_\eta \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \langle\chi\rangle = v_\chi \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \qquad \langle\xi\rangle = v_\xi \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$
$$\langle\rho\rangle = v_\rho, \qquad \langle\sigma\rangle = v_\sigma, \qquad \langle\vartheta\rangle = v_\vartheta, \tag{3.5}$$

for flavons in Φ^e sector and

$$\langle\phi\rangle = v_\phi \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle\Delta\rangle = v_\Delta \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \langle\zeta\rangle = v_\zeta \tag{3.6}$$

for flavons in Φ^ν sector. The VEVs of φ, η, χ and ξ break S_4 completely, since acting on the vacua with T or T^2 the directions of them are invariant except an overall phase, while those of ϕ and Δ are invariant under the four elements 1, S^2, TST and $TSTS^2$. Moreover the order of magnitude for the VEVs $\langle\Phi^e\rangle$ and $\langle\Phi^\nu\rangle$ are taken as $\lambda_c^2\Lambda$ and $\lambda_c^3\Lambda$, respectively. The reason for the constrains on the order of the VEVs is that they should be responsible for the strong mass hierarchies of charged fermions. The small expansion parameters $\epsilon = \frac{\langle\Phi^e\rangle}{\Lambda}$ and $\delta = \frac{\langle\Phi^\nu\rangle}{\Lambda}$ are used in the following discussions.

3.1 Neutrino

The right-handed neutrinos are SU(5) singlet **1** in the model, thus the light neutrino masses are only generated through type-I seesaw mechanism

$$m_\nu = -m_D^T M_M^{-1} m_D \tag{3.7}$$

where the m_D and M_M are Dirac and Majorana mass matrices respectively. The two matrices are derived from the superpotential invariant under the flavor symmetry. Concretely the superpotential in neutrino sector is as follows

$$w_\nu = \frac{y_{\nu_1}}{\Lambda}(FN^c)_{3_1}\phi H_5 + \frac{y_{\nu_2}}{\Lambda}(FN^c)_2\Delta H_5 + \frac{1}{2}MN^cN^c \tag{3.8}$$

The first two terms contribute to Dirac masses and the last one is Majorana righted-handed neutrinos mass. The Tri-Bimaximal mixing is reproduced by the vacuum alignments of scalar fields ϕ and Δ, see eq. (4.14). Note that $\langle\phi\rangle$ and $\langle\Delta\rangle$ are invariant by acting the elements 1, S^2, TST and $TSTS^2$ on them, hence the flavor symmetry is broken down to the Klein four subgroup. After Electroweak and flavor symmetry breaking as Higgs field and flavons developing their VEVs, neutrinos will gain their masses. For Dirac neutrino mass matrix at LO we have

$$m_D = \frac{y_{\nu_1} v_\phi v_5}{\Lambda}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \frac{y_{\nu_2} v_\Delta v_5}{\Lambda}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \tag{3.9}$$

and Majorana mass matrix is

$$M_M = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix} \tag{3.10}$$

The eigenvalues of Majorana mass matrix M_M can be diagonalized by unitary transformation

$$U_R^T M_M U_R = \mathrm{diag}(M, M, M), \quad U_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha}/\sqrt{2} & -ie^{i\alpha}/\sqrt{2} \\ 0 & e^{-i\alpha}/\sqrt{2} & ie^{-i\alpha}/\sqrt{2} \end{pmatrix} \tag{3.11}$$

where α is a phase parameter. All three right-handed neutrinos are degenerate with mass equal to M, the unitary transformation cannot be solely determined.

Using eq. (3.7), the derived light neutrino mass matrix can be diagonalized by the transformation

$$m_\nu^{\mathrm{diag}} = U_\nu^T m_\nu U_\nu = \mathrm{Diag}(m_1, m_2, m_3) \tag{3.12}$$

where the light neutrino masses m_1, m_2, m_3 are

$$m_1 = \left| -\frac{(3a-b)^2}{M} \right| \frac{v_5^2}{\Lambda^2}, \quad m_2 = \left| -\frac{4b^2}{M} \right| \frac{v_5^2}{\Lambda^2}, \quad m_3 = \left| \frac{(3a+b)^2}{M} \right| \frac{v_5^2}{\Lambda^2} \tag{3.13}$$

in which $a = y_{\nu_1} v_\phi$, $b = y_{\nu_2} v_\Delta$. While the unitary matrix U_ν in eq. (3.12) is given by

$$U_\nu = U_{\mathrm{TB}} P_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} e^{i\vartheta_1/2} & 0 & 0 \\ 0 & e^{i\vartheta_2/2} & 0 \\ 0 & 0 & e^{i\vartheta_3/2} \end{pmatrix} \tag{3.14}$$

Thus the famous TBM mixing matrix is obtained at LO exactly. The phases $\vartheta_i, i = 1, 2, 3$ can be easily obtained as following

$$\vartheta_1 = \arg\left(-\frac{(3a-b)^2}{M}\frac{v_5^2}{\Lambda^2}\right), \quad \vartheta_2 = \arg\left(-\frac{4b^2}{M}\frac{v_5^2}{\Lambda^2}\right), \quad \vartheta_3 = \arg\left(\frac{(3a+b)^2}{M}\frac{v_5^2}{\Lambda^2}\right) \tag{3.15}$$

Note that the lepton PMNS mixing matrix is often parameterized by the standard PDG form as

$$U_\ell = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} e^{-i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{-i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.16}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with $\theta_{ij} \in [0, \pi/2]$. The Dirac CP violating phase δ_{13}, the two Majorana CP violating phases α_1 and α_2 are all permitted to vary in the period of $0 \sim 2\pi$. In the following we shall identify the two Majorana phases as $\alpha_i = \vartheta_3 - \vartheta_i$ and express them in terms of the lightest neutrino mass.

Here let us leave the angles and phases for a moment and estimate the scale of M and the range of lightest neutrino mass in both hierarchies. The mass square differences have been determined to high precision in several global neutrino data fits, such as [101–103]

$$\Delta m^2_{\rm sol} = 7.50^{+0.19}_{-0.197} \times 10^{-5}{\rm eV}^2, \quad \begin{cases} \Delta m^2_{\rm atm} = 2.457^{+0.047}_{-0.047} \times 10^{-3}{\rm eV}^2 & ({\rm NH}) \\ \Delta m^2_{\rm atm} = -2.449^{+0.048}_{-0.047} \times 10^{-3}{\rm eV}^2 & ({\rm IH}) \end{cases} \tag{3.17}$$

where NH (IH) stands for normal (inverted) hierarchy of mass spectrum. Despite of the discrepancy of specific values and their accuracy, another works of global fit data can be seen in ref. [104–107]. The magnitude of masses m_i can be roughly estimated around 10^{-2} eV–10^{-1} eV, and v_5 is electroweak scale $\sim 10^2$ GeV, then generally the scale of M will be

$$M \sim 10^{11 \div 12}\,{\rm GeV}. \tag{3.18}$$

The parameters a and b are assumed to be of order λ_c^3, we can define the ratio as

$$\frac{a}{b} = Re^{i\Theta}. \tag{3.19}$$

then we can express the real parameters R and Θ in terms of the light neutrino masses in eq. (3.13) as

$$R = \frac{1}{3}\sqrt{2\left(\frac{m_3}{m_2} + \frac{m_1}{m_2}\right) - 1}, \qquad \cos\Theta = \frac{\frac{m_3}{m_2} - \frac{m_1}{m_2}}{\sqrt{2\left(\frac{m_3}{m_2} + \frac{m_1}{m_2}\right) - 1}}. \tag{3.20}$$

Taking into account the experimental values of two mass differences, cf. eq. (3.17), we have only one free parameter left, which can be chosen to be the lightest neutrino mass (m_1 in NH or m_3 in IH) for convenient. Imposing the constraint $|\cos\Theta| \le 1$, one could obtain the limits for the lightest neutrino masses as

$$\begin{aligned} m_1 &\ge 0.011{\rm eV}, \quad {\rm NH} \\ m_3 &\ge 0.028{\rm eV}, \quad {\rm IH} \end{aligned} \tag{3.21}$$

where only the best fit values are used in the estimation.

The Majorana phases in the PDG standard parameterization are naively defined as

$$\alpha_1 = \vartheta_3 - \vartheta_1, \quad \alpha_2 = \vartheta_3 - \vartheta_2, \tag{3.22}$$

inserting the expressions of ϑ_i, R and Θ, cf. eq. (3.15) and eq. (3.20), respectively, yield

$$\begin{aligned} \sin\alpha_1 &= \frac{12R(1-9R^2)\sin\Theta}{(1+9R^2)^2 - 36R^2\cos^2\Theta}, \qquad & \cos\alpha_1 &= \frac{(1-9R^2)^2 - 36R^2\sin^2\Theta}{(1+9R^2)^2 - 36R^2\cos^2\Theta} \\ \sin\alpha_2 &= \frac{6R\sin\Theta(1+3R\cos\Theta)}{1+9R^2+6R\cos\Theta}, \qquad & \cos\alpha_2 &= \frac{1+9R^2\cos2\Theta + 6R\cos\Theta}{1+9R^2+6R\cos\Theta}. \end{aligned} \tag{3.23}$$

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The two Majorana phases α_1 and α_2 can take two different sets of values in principle which corresponding to $\sin\Theta > 0$ and $\sin\Theta < 0$, respectively. The reason is that the neutrino mass order which can be either NH or IH determines the sigh of $\cos\Theta$. The Dirac CP phase δ_{13} is undetermined for the vanishing θ_{13} in TBM mixing.

Besides the operators in eq. (3.8), it is worth to consider effective operators, i.e., the higher dimensional Weinberg operators [110] would also contribute to neutrino masses. In this model the effective operators for both Dirac and Majorana mass terms are[1]

$$\begin{aligned} W_\nu^{\text{eff}} &= \frac{y_1}{\Lambda_W} FFH_5H_5 + \sum_{i=1}^{3} \frac{y_i^{(1_1)}}{\Lambda_W^3} FFH_5H_5\mathcal{O}_i^{1_1} + \sum_{i=1}^{2} \frac{y_i^{(2)}}{\Lambda_W^3} FFH_5H_5\mathcal{O}_i^{(2)} \\ &+ \sum_{i=1}^{2} \frac{y_i^{(3_1)}}{\Lambda_W^3} FFH_5H_5\mathcal{O}_i^{(3_1)} + \frac{y_2}{\Lambda_W^4} FFH_{45}H_{45}(\phi\Delta)_{3_2}\zeta, \end{aligned} \tag{3.24}$$

where the operators $\mathcal{O}_i$ represent the following specific S_4 contractions of flavons in Φ^ν

$$\mathcal{O}_i^{1_1} = \{(\phi\phi)_{1_1}, (\Delta\Delta)_{1_1}, \zeta\zeta\}, \qquad \mathcal{O}_i^{2} = \{(\phi\phi)_2, (\Delta\Delta)_2\}, \qquad \mathcal{O}_i^{3_1} = \{(\phi\phi)_{3_1}, (\phi\Delta)_{3_1}\} \tag{3.25}$$

With the VEVs of Higgs fields, the operators generate the neutrino mass terms, in unit of $\frac{v_5^2}{\Lambda_W}$ as follows

$$m_W = y^{(1_1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + y^{(2)} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + y^{(3_1)} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \tag{3.26}$$

in which the coefficients are

$$\begin{aligned} y^{(1_1)} &= y_1 + 3y_1^{(1_1)} \frac{v_\phi^2}{\Lambda_W^2} + 2y_2^{(1_1)} \frac{v_\Delta^2}{\Lambda_W^2} + y_3^{(1_1)} \frac{v_\zeta^2}{\Lambda_W^2}, \\ y^{(2)} &= 3y_1^{(2)} \frac{v_\phi^2}{\Lambda_W^2} + \frac{v_\Delta^2}{\Lambda_W^2}, \quad y^{(3_1)} = 2y_2^{(3_1)} \frac{v_\phi v_\Delta}{\Lambda_W^2} \end{aligned} \tag{3.27}$$

One can realize the structure of m_W is exactly the same as that of m_D in eq. (3.9) and of M_M in eq. (3.10) combined, hence it can be exactly diagonalized by TBM mixing matrix

$$m_W^{\text{diag}} = U_{\text{TB}}^T m_W U_{\text{TB}} = \text{Diag}(m_{W1}, m_{W2}, m_{W3}), \tag{3.28}$$

where the light effective neutrino masses m_{W1}, m_{W2} and m_{W3} come from the Weinberg operators, they are given by

$$m_{W1} = y^{(1_1)} - y^{(2)} + 3y^{(3_1)}, \quad m_{W2} = y^{(1_1)} + 2y^{(2)}, \quad m_{W3} = -y^{(1_1)} + y^{(2)} + 3y^{(3_1)} \tag{3.29}$$

Obviously we can compare the relative magnitude between m_ν^{diag} and m_W^{diag} with the assumption that all couplings, $y_{\nu_{1,2}}$, and y^c, are of order one, then the ratio could be

$$\frac{m_{Wi}}{m_i} \sim \frac{M\Lambda^2}{\Lambda_W v_\phi^2} \sim 10^4 \frac{M}{\Lambda_W} \sim \frac{\Lambda_{\text{GUT}}}{\Lambda_W} \tag{3.30}$$

[1]The operators FFH_5H_5 and $FFH_{45}H_{45}$ represent $(F_i)_\alpha(F_j)_\beta H_5^\alpha H_5^\beta$ and $(F_i)_\alpha(F_j)_\beta(H_{45})_\delta^{\gamma\alpha}(H_{45})_\gamma^{\delta\beta}$ respectively, where the Greek indices denote the SU(5) tensor contractions, the Latin indices are the fermion generations or say contractions in S_4 space.

The contribution to neutrino mass would be larger than the seesaw one if the Weinberg operator has cutoff $\Lambda_W \sim \Lambda_{\rm GUT}$. In order to avoid the problem we will require $\Lambda_W \gg \Lambda_{\rm GUT}$, such that $\Lambda_W \sim \Lambda_{\rm Planck}$, then the 5-dimensional effective operator can be neglected.

3.2 Up-type quarks

The masses of up-type quarks are generated by S_4 symmetry breaking in the invariant superpotential at LO. The enormous mass hierarchies among up-type quarks should be guaranteed by the VEVs of scalar flavons. At LO the invariant superpotential under the whole symmetry groups is simply as[2]

$$\begin{aligned} w_U = {} & y_t T_3 T_3 H_5 + \frac{y_c}{\Lambda^2} T_2 T_2 \sigma \vartheta H_5 + \frac{y_{ct}}{\Lambda} T_2 T_3 \sigma H_{45} + \frac{y_u}{\Lambda^3} T_1 T_1 H_5 \eta \Delta \zeta \\ & + \sum_{i=1}^{3} \frac{y'_{ti}}{\Lambda^2} T_3 T_3 H_5 \mathcal{O}_i^{U1} + \sum_{i=1}^{3} \frac{y'_{cti}}{\Lambda^3} T_2 T_3 H_{45} \mathcal{O}_i^{U2} \end{aligned} \tag{3.31}$$

where the operators read

$$\mathcal{O}_i^{U1} = \{\zeta\zeta, \Delta\Delta, \phi\phi\}, \quad \mathcal{O}_i^{U2} = \{\sigma\zeta\zeta, \sigma\Delta\Delta, \sigma\phi\phi\} \tag{3.32}$$

Then after the scalar fields develop their VEVs in eqs. (4.3) and (4.6) and gauge symmetry breaking, the mass matrix of up-type quarks can be written as

$$M_U = \begin{pmatrix} -8y_u \frac{v_\eta v_\Delta v_\zeta}{\Lambda^3} v_5 & 0 & 0 \\ 0 & 8y_c \frac{v_\sigma v_\vartheta}{\Lambda^2} v_5 & 8y_{ct} \frac{v_\sigma}{\Lambda} v_{45} \\ 0 & -8y_{ct} \frac{v_\sigma}{\Lambda} v_{45} & 8y_t v_5 \end{pmatrix} + 8 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y'_{ct}\epsilon\delta^2 v_{45} \\ 0 & -y'_{ct}\epsilon\delta^2 v_{45} & y'_t \delta^2 v_5 \end{pmatrix} \tag{3.33}$$

Then mass matrix can be diagonalized by the bi-unitary transformation

$$m_U = U_R^\dagger M_U U_L = \mathrm{Diag}(m_u, m_c, m_t) \tag{3.34}$$

in which the mass eigenvalues are

$$m_u = \left| -8y_u \frac{v_\eta v_\Delta v_\zeta}{\Lambda^3} v_5 \right|, \quad m_c = \left| 8y_c \frac{v_\sigma v_\vartheta}{\Lambda^2} v_5 + 8 \frac{y_{ct}^2}{y_t} \frac{v_\sigma^2}{\Lambda^2} \frac{v_{45}^2}{v_5} \right|, \quad m_t = |8 y_t v_5| \tag{3.35}$$

where the small contributions from the second matrix in eq. (3.33) is safely dropped or say reabsorbed into the redefinition of the couplings y_t and y_{ct}. In fact all the correctional contributions to the (33) element can be reabsorbed into the y_t whose order of magnitude is not changed. We can summarize that all the masses of up-type quarks are obtained at LO, particularly the top quark mass is produced at tree level. The mass hierarchy between charm and top quark is obtained given that the v_σ and v_ϑ of order $\lambda_c^2\Lambda$. The up quark is also produced the correct order by means of $v_\eta \sim \lambda_c^2\Lambda$ and $v_{\Delta,\zeta} \sim \lambda_c^3\Lambda$. The mass

[2]The product $T_i T_j H_5$ denotes $\varepsilon_{\alpha\beta\gamma\rho\sigma} T_i^{\alpha\beta} T_j^{\gamma\rho} H_5^\sigma$, and $T_i T_j H_{45}$ denotes $\varepsilon_{\alpha\beta\gamma\rho\sigma} T_i^{\alpha\beta} T_j^{\zeta\gamma} (H_{45})_\zeta^{\rho\sigma}$, where the Greek indices indicates the SU(5) tensor contractions and, Latin indices $i, j = 1, 2, 3$, are family indices. Here we don't write the S_4 contractions because all contractions in S_4 space should be first reduced to that in SU(5) space. The SU(5) tensor contractions are more fundamental ones for the calculations, the Clebsch-Gordan coefficients in eq. (3.33) are mainly determined by those SU(5) tensor contractions.

hierarchies are obtained as usual: $m_u : m_c : m_t = \lambda_c^8 : \lambda_c^4 : 1$. Due to the quantity λ_c^8 is only comprised of the combination $\epsilon\delta^2$ rather than ϵ^4, we can conclude that the hierarchical VEVs of Φ^e and Φ^ν are essential in the model, as declared in section 2.

After diagonalize the mass matrix (3.33), one can note that the mixing of up-type quarks only exists between charm quark and top quark, which is an experimental acceptable feature of our model. Thus the form of the resulting mixing matrix U_L is rather simple

$$U_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & S_{23}^u \\ 0 & -S_{23}^{u*} & 1 \end{pmatrix} \tag{3.36}$$

with the mixing angles

$$S_{12}^u = S_{13}^u = 0, \qquad S_{23}^u = -\left(\frac{y_{ct}}{y_t}\frac{v_\sigma}{\Lambda}\frac{v_{45}}{v_5}\right)^*, \tag{3.37}$$

where S_{ij}^u is the sine of mixing angles and $C_{ij}^u \simeq 1$ is assumed. In the following section we may aware of the (23) and (32) elements of CKM mixing matrix are totally determined by S_{23}^u.

3.3 Down-type quarks and charged leptons

The matter fields appeared in the superpotential w_U in eq. (3.31) which gives rise to the masses of up-type quarks only include the SU(5) **10**-dimension representational T_i ($i = 1, 2, 3$), while down-type quarks and charged leptons are not the case. The down-type quark fields (and charged lepton fields) and their conjugate fields are assigned to be **10** and $\bar{\mathbf{5}}$ representations of SU(5), respectively. Consequently the superpotential of down-type quarks and charged leptons is comprised of T_i and F together with the down-type Higgs fields $H_{\overline{5},\overline{45}}$ and the flavons. In addition the adjoint field H_{24} plays a important role to gain the novel CG factors that can lead to a desired GUT relation between masses and mixing angle, as elucidated in section 2. To be specific the LO superpotential which gives rise to the masses of down-type quarks and charged leptons is[3]

$$\begin{aligned} w_D = {} & \frac{y_b}{\Lambda}T_3F\varphi H_{\bar{5}} + \sum_{i=1}^{2}\frac{y_{si}}{\Lambda^2}T_2F\mathcal{O}_i^{D1}H_{\overline{45}} + \frac{y_{bd}}{\Lambda^2}T_1F\varphi\sigma H_{\overline{45}} + \frac{y_s'}{\Lambda^3}T_2F(\varphi\eta)_{3_2}\sigma H_{\overline{45}} \\ & + \sum_{i=1}^{2}\frac{f_{di}}{\langle H_{24}\rangle^3}T_1F\mathcal{O}_i^{D2}H_{\bar{5}} + \sum_{i=1}^{3}\frac{g_{di}}{\Lambda^2\langle H_{24}\rangle}T_2F\mathcal{O}_i^{D3}H_{\bar{5}} + \sum_{i=1}^{6}\frac{h_{di}}{\Lambda^3}T_3F\mathcal{O}_i^{D4}H_{\bar{5}} + \cdots \end{aligned} \tag{3.38}$$

where dots stand for higher order operators, and the operators $\mathcal{O}^D$ are

$$\begin{aligned} &\mathcal{O}_i^{D1} = \{\chi\sigma, \xi\zeta\}, && \mathcal{O}_i^{D2} = \{\chi\xi\rho, \chi\xi\xi\}, \\ &\mathcal{O}_i^{D3} = \{\phi\phi\phi, \phi\phi\Delta, \phi\Delta\Delta\}, && \mathcal{O}_i^{D4} = \{\varphi\phi\phi, \varphi\phi\Delta, \varphi\Delta\Delta, \varphi\zeta\zeta, \eta\phi\phi, \eta\Delta\phi\}. \end{aligned} \tag{3.39}$$

[3] Similarly it is easy to write the basic contractions of these operators in SU(5) space: $T_iF_jH_{\bar{5}} = T_i^{\alpha\beta}(F_j)_\alpha(H_{\bar{5}})_\beta$ and, $T_iF_jH_{\overline{45}} = T_i^{\alpha\beta}(F_j)_\gamma(H_{\overline{45}})^\gamma_{\alpha\beta}$. The index structures of the contractions can be found, e.g., in [111].

The operators involved $\mathcal{O}^D$ in eq. (3.38) should include all possible independent S_4 contractions. The down-type quarks and charged leptons in the superpotential eq. (3.38) would obtain their masses as the Higgs fields and flavon fields developing their VEVs after the symmetries are broken. Note that the operators involved $\mathcal{O}_i^{D3}$ have vanishing contribution to the entries of Yukawa matrix. With the VEVs of the flavons and Higgses, the mass matrix of down-type quarks is immediately derived as follows

$$M_D = \begin{pmatrix} 0 & y^d_{12}\epsilon\delta v_{\overline{45}} & y^d_{13}\epsilon\delta^2 v_{\bar{5}} \\ y^d_{21}\epsilon^3 v_{\bar{5}} & (y^d_{22}+y^{d\prime}_{22}\epsilon)\epsilon^2 v_{\overline{45}} & y^d_{23}\epsilon\delta^2 v_{\bar{5}} \\ y^d_{31}\epsilon^2 v_{\overline{45}} & 0 & y^d_{33}\epsilon v_{\bar{5}} + y^{d\prime}_{33}\epsilon\delta^2 v_{\bar{5}} \end{pmatrix} \tag{3.40}$$

where the coefficients y^d_{ij} (i, j=1, 2, 3) and those with primes are linear combinations of LO coefficients. The (21) element implies $v_{\Phi^e}/\langle H_{24}\rangle \sim \epsilon$, or equivalently $\langle H_{24}\rangle \sim \Lambda$,[4] which is an essential condition to obtain the proper order of magnitude for the masses of down quark and electron, and hence for the hierarchies of all the masses. Similarly the mass matrix of charged leptons which is equivalent to the transposed M_D is given as

$$M_\ell = \begin{pmatrix} 0 & -\frac{8}{27}y^d_{21}\epsilon^3 v_{\bar{5}} & -3y^d_{31}\epsilon^2 v_{\overline{45}} \\ -3y^d_{12}\epsilon\delta v_{\overline{45}} & -3(y^d_{22}+y^{d\prime}_{22}\epsilon)\epsilon^2 v_{\overline{45}} & 0 \\ y^d_{13}\epsilon\delta^2 v_{\bar{5}} & y^d_{23}\epsilon\delta^2 v_{\bar{5}} & y^d_{33}\epsilon v_{\bar{5}} + y^{d\prime}_{33}\epsilon\delta^2 v_{\bar{5}} \end{pmatrix} \tag{3.41}$$

The CG coefficients which manifest in the entries of the mass matrices are determined by the way of the tensors contracted in SU(5) space. The two mass matrices M_D and M_ℓ can be diagonalized by the similar bi-unitary transformations as in up quark sector

$$V_R^{D\dagger} M_D D_L = \mathrm{diag}(m_d, m_s, m_b), \quad V_R^{\ell\dagger} M_\ell V_L^\ell = \mathrm{diag}(m_e, m_\mu, m_\tau). \tag{3.42}$$

The mass eigenvalues of down type quarks are

$$m_d \simeq \left| -\frac{y^d_{21}y^d_{12}}{y^d_{22}}\epsilon^2\delta v_{\bar{5}} \right|, \quad m_s \simeq \left| (y^d_{22}+y^{d\prime}_{22}\epsilon)\epsilon^2 v_{\overline{45}} + \frac{y^d_{21}y^d_{12}}{y^d_{22}}\epsilon^2\delta v_{\bar{5}} \right|, \quad m_b \simeq |y^d_{33}\epsilon v_{\overline{45}}| \tag{3.43}$$

and those of charged leptons are

$$m_e \simeq \left| \frac{8}{27}\frac{y^d_{21}y^d_{12}}{y^d_{22}}\epsilon^2\delta v_{\bar{5}} \right|, \quad m_\mu \simeq \left| -3(y^d_{22}+y^{d\prime}_{22}\epsilon)\epsilon^2 v_{\overline{45}} - \frac{8y^d_{21}y^d_{12}}{27y^d_{22}}\epsilon^2\delta v_{\bar{5}} \right|, \quad m_\tau \simeq |y^d_{33}\epsilon v_{\bar{5}}| \tag{3.44}$$

From the mass expressions in eq. (3.43) and eq. (3.44), one can easily find that bottom quark and tau lepton have the same mass, and the mass of muon is three times of that of strange quark, and the mass of electron is $\frac{8}{27}$ of that of down quark

$$m_\tau \simeq m_b, \qquad m_\mu \simeq 3m_s, \qquad m_e \simeq \frac{8}{27}m_d, \tag{3.45}$$

[4]The adjoint field H_{24} has the VEV along the direction $\langle H_{24}\rangle = \sqrt{\frac{2}{15}}v_{24}\mathrm{diag}(1,1,1,-\frac{3}{2},-\frac{3}{2})$, which is responsible for the broken of SU(5) GUT symmetry down to the standard model symmetry $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

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The bottom-tau unification and Georgi-Jarlskog relation [95] are produced in the model, and a new novel mass ratio $\frac{m_e}{m_d}$ which is favored in phenomenology is obtained. The above mass relations between down-type quarks and charged leptons give rise to the combined relation $\frac{m_\mu}{m_s}\frac{m_d}{m_e} \simeq 10.1$, which is well within the 1σ range discussed in [112], i.e., the double ratio of Yukawa couplings at the scale $M_{\rm GUT}$

$$\frac{y_\mu}{y_s}\frac{y_d}{y_e} \approx 10.7 \pm^{1.8}_{0.8}\,. \tag{3.46}$$

In contrast, the double ratio in the distinguished original George-Jarlskog relation [95], which implies $\frac{y_\mu}{y_s}\frac{y_d}{y_e} = 9$, deviates from the phenomenological favored result more than 2σ.

The unitary transformation matrices D_L and V_L^ℓ are approximately as

$$D_L = \begin{pmatrix} 1 & \left(\frac{y^d_{21}}{y^d_{22}}\frac{v_{\bar 5}}{v_{\overline{45}}}\epsilon\right)^* & \left(\frac{y^d_{31}}{y^d_{33}}\frac{v_{\overline{45}}}{v_{\bar 5}}\epsilon\right)^* \\ -\frac{y^d_{21}}{y^d_{22}}\frac{v_{\bar 5}}{v_{\overline{45}}}\epsilon & 1 & 0 \\ -\frac{y^d_{31}}{y^d_{33}}\frac{v_{\overline{45}}}{v_{\bar 5}}\epsilon & \left(\frac{y^{d*}_{21}}{y^{d*}_{22}}\frac{y^d_{31}}{y^d_{33}}\right)|\epsilon|^2 & 1 \end{pmatrix} \tag{3.47}$$

$$V_L^\ell = \begin{pmatrix} 1 & \left(\frac{y^d_{12}}{y^d_{22}}\frac{\delta}{\epsilon}\right)^* & 0 \\ -\frac{y^d_{12}}{y^d_{22}}\frac{\delta}{\epsilon} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.48}$$

The complete quark mixing matrix $V_{\rm CKM}$ is composed of mixing matrices of both up-type quark and down-type quark sector

$$V_{\rm CKM} = U_L^\dagger D_L \tag{3.49}$$

then we can directly get all elements of CKM matrix

$$\begin{aligned} &V_{ud} \simeq V_{cs} \simeq V_{tb} \simeq 1, && V^*_{us} \simeq -V_{cd} \simeq \frac{y^d_{21}}{y^d_{22}}\frac{v_{\bar 5}}{v_{\overline{45}}}\epsilon, \qquad V^*_{ub} \simeq \frac{y^d_{31}}{y^d_{33}}\frac{v_{\overline{45}}}{v_{\bar 5}}\epsilon \\ &V_{td} \simeq -\frac{y^d_{31}}{y^d_{33}}\frac{v_{\overline{45}}}{v_{\bar 5}}\epsilon - \frac{y^d_{21}}{y^d_{22}}\frac{v_{\bar 5}}{v_{\overline{45}}}\frac{y_{ct}}{y_t}\frac{v_\sigma}{\Lambda}\frac{v_{45}}{v_5}\epsilon, && V^*_{cb} \simeq -V_{ts} \simeq \frac{y_{ct}}{y_t}\frac{v_\sigma}{\Lambda}\frac{v_{45}}{v_5} \end{aligned} \tag{3.50}$$

It is an experimental constrains that V_{us} and V_{cd} are Cabibbo angle λ_c, V_{ub} and V_{td} are of order λ_c^3, which all demands a fine tuning between $v_{\overline{45}}$ and $v_{\bar 5}$: $v_{\overline{45}} \sim \lambda_c v_{\bar 5}$. Adding this condition, we can easily check that the quark CKM mixing matrix is produced correctly. The Cabibbo angle are determined by the mixing between the first and second family down-type quarks, the parameters y^d_{21} and y^d_{22} are of order one was assumed. V_{cb} and V_{ts} are determined by mixing between second and third generation left-handed up quarks .

Similarly the resulting lepton mixing matrix $U_{\rm PMNS}$ is written as

$$U_{\rm PMNS} = V_L^{\ell\dagger} U_\nu, \quad U_\nu = U_{\rm TB} P_\nu \tag{3.51}$$

The desired CKM-like mixing matrix V_L^ℓ in eq. (3.48) implies a large mixing angle between the first and the second generation of charged leptons, and it will remarkably change the lepton mixing, although the TBM mixing is exactly produced in neutrino sector. After simple straight calculation, we arrive at the three leptonic mixing angles $\theta_{ij}^{\rm PMNS}$ at LO as following

$$\begin{aligned}
\sin^2\theta_{12}^{\rm PMNS} &= \frac{|(U_{\rm PMNS})_{e2}|^2}{1-|(U_{\rm PMNS})_{e3}|^2} \\
&= \frac{1}{3} - \frac{2}{3}Re\left(\frac{y_{12}^d}{y_{22}^d}\frac{\delta}{\epsilon}\right) + \frac{1}{2}\left|\frac{y_{12}^d}{y_{22}^d}\frac{\delta}{\epsilon}\right|^2 - \frac{1}{3}\left|\frac{y_{12}^d}{y_{22}^d}\frac{\delta}{\epsilon}\right|^2 Re\left(\frac{y_{12}^d}{y_{22}^d}\frac{\delta}{\epsilon}\right) \\
\sin^2\theta_{23}^{\rm PMNS} &= \frac{|(U_{\rm PMNS})_{\mu3}|^2}{1-|(U_{\rm PMNS})_{e3}|^2} = \frac{1}{2}\left(1+\frac{1}{2}\left|\frac{y_{12}^d}{y_{22}^d}\frac{\delta}{\epsilon}\right|^2\right) \\
\sin\theta_{13}^{\rm PMNS} &= |(U_{\rm PMNS})_{e3}| = \frac{1}{\sqrt{2}}\left|\frac{y_{12}^d}{y_{22}^d}\frac{\delta}{\epsilon}\right|
\end{aligned} \tag{3.52}$$

As elucidated in section 2, the ratio of CG coefficients $|\frac{c_c}{c_b}| \sim \mathcal{O}(2/\lambda_c)$ is required to obtain the large angle $\theta_{12}^e \sim \lambda_c$ in the GUT relation with the phenomenological mass ratio $\frac{m_e}{m_\mu}$ and θ_{12}^d. In the present model the value of the ratio $|\frac{c_c}{c_b}|$ is $\frac{81}{8}$, which satisfies the requirement of order $\mathcal{O}(2/\lambda_c)$. Then we can conclude that the mixing matrix V_L^ℓ is accurate enough to hold $\theta_{12}^e \sim \lambda_c$ and hence the empirical relation $\theta_{13}^{\rm PMNS} \simeq \lambda_c/\sqrt{2}$ is obtained.

The large $\theta_{13}^{\rm PMNS}$ arises from the contribution of charged lepton sector, and $\theta_{12}^{\rm PMNS}$ prominent deviates from its TBM value in the case. Since the reactor experiments have showed that lepton mixing angle $\theta_{13}^{\rm PMNS}$ are about $\lambda_c/\sqrt{2}$, which meant $|\frac{y_{12}^d}{y_{22}^d}\frac{\delta}{\epsilon}| \sim \lambda_c$, then the deviations of three leptonic mixing angles from their TBM values are roughly estimated as follows

$$\sin\theta_{13}^{\rm PMNS} \sim \frac{\lambda_c}{\sqrt{2}}, \qquad \left|\sin^2\theta_{12}^{\rm PMNS} - \frac{1}{3}\right| \sim \frac{2}{3}\lambda_c, \qquad \left|\sin^2\theta_{23}^{\rm PMNS} - \frac{1}{2}\right| \sim \frac{\lambda_c^2}{4} \tag{3.53}$$

The relations are compatible with leptonic mixing sum rules [89]. Recall that the experimental value of $\theta_{12}^{\rm PMNS}$ [101–103] is very close to TBM value, the large departure is seemingly unsuitable. Setting the expansion parameters ϵ and δ to be positive real numbers for simplicity, the problem could be settled by taking into account of a Dirac CP violating phase, which is in fact the complex phase, denoted by ϕ_{12}, of the ratio y_{12}^d/y_{22}^d. We remind the readers that ϕ_{12} is in fact determined by $\arg(y_{12}^d\delta/y_{22}^d\epsilon)$, since the expansion parameters $\delta = \langle\Phi^\nu\rangle/\Lambda$ and $\epsilon = \langle\Phi^e\rangle/\Lambda$ are in general complex numbers as y_{ij}^ds. The phases of δ and ϵ, however, can be incorporated by the phases of y_{ij}^ds. Ignoring the higher order deviations, the deviation of angle $\theta_{12}^{\rm PMNS}$ from the TBM prediction is approximately expressed as following

$$\sin^2\theta_{12}^{\rm PMNS} - \frac{1}{3} \sim -\frac{2\sqrt{2}}{3}\Big|(U_{\rm PMNS})_{e3}\Big|\cos\phi_{12}, \qquad \phi_{12} = \arg\left(\frac{y_{12}^d}{y_{22}^d}\right) \tag{3.54}$$

The correlation has been given in [89] as mentioned before, and similar result with minus sign difference (because of different diagonalization conventions of fermion mass matrix) has been obtained in ref. [58]. In order to be consistent with the TBM value, the phase ϕ_{12} should be around $\pi/2$ or $-\pi/2$ so that the sizable departure vanishes, or at least decreases to be of order λ_c^2. The detailed study about the link between CP violation and charged lepton corrections to mixing angles is beyond the scope of the present work, one may refer [108] for example.

4 Vacuum alignment

The vacuum alignment would be discussed as the natural solution of the scalar potential. The problem can be solved by so-called supersymmetric driving field method introduced by Altarelli and Feruglio in ref. [100]. At LO, the superpotential of driving fields, which is invariant under the flavor symmetry $S_4 \times Z_4 \times Z_6 \times Z_5 \times Z_2$, is given by $w_d = w_d^e(\varphi_0, \chi_0, \eta_0, \sigma_0, \xi_0) + w_d^\nu(\phi_0, \Delta_0, \zeta_0)$ where

$$\begin{aligned}
w_d = \; & g_1\varphi_0\varphi\varphi + g_2(\varphi_0\varphi)_2\eta + h_1\eta_0(\varphi\varphi)_{1_1} + h_2\eta_0(\eta\eta)_{1_1} + g_3M_\chi\chi_0\chi + g_4\chi_0(\varphi\eta)_{3_2} \\
& + q_1\sigma_0\sigma\sigma + q_2\sigma_0\vartheta\vartheta + r_1\xi_0\xi\xi + r_2\xi_0\xi\rho \\
& + f_1\phi_0(\phi\Delta)_{3_2} + f_2\Delta_0(\phi\phi)_2 + f_3\Delta_0\Delta\Delta + f_4\zeta_0\zeta\zeta + f_5\zeta_0(\phi\phi)_{1_1} + f_6\zeta_0(\Delta\Delta)_{1_1}.
\end{aligned} \tag{4.1}$$

The vacuum alignments of all flavons in Φ^e and Φ^ν are determined by deriving w_d with respect to each component of the driving fields Φ_0 in SUSY limit. After minimized the derivative equations and solved each unknown component of all flavons, the VEVs structures of the flavons can be obtained. Usually the solutions are not uniquely determined, we should choose one set by taking into account some constrained conditions. The detailed minimization equations of flavons φ, η and χ

$$\begin{aligned}
\frac{\partial w_d}{\partial \varphi_{01}} &= 2g_1(\varphi_1^2 - \varphi_2\varphi_3) + g_2(\eta_1\varphi_2 + \eta_2\varphi_3) = 0 \\
\frac{\partial w_d}{\partial \varphi_{02}} &= 2g_1(\varphi_2^2 - \varphi_1\varphi_3) + g_2(\eta_1\varphi_1 + \eta_2\varphi_2) = 0 \\
\frac{\partial w_d}{\partial \varphi_{03}} &= 2g_1(\varphi_3^2 - \varphi_1\varphi_2) + g_2(\eta_1\varphi_3 + \eta_2\varphi_1) = 0 \\
\frac{\partial w_d}{\partial \chi_{01}} &= g_3M_\chi\chi_1 + g_4(\varphi_3\eta_2 - \varphi_2\eta_1) = 0 \\
\frac{\partial w_d}{\partial \chi_{02}} &= g_3M_\chi\chi_3 + g_4(\varphi_2\eta_2 - \varphi_1\eta_1) = 0 \\
\frac{\partial w_d}{\partial \chi_{03}} &= g_3M_\chi\chi_2 + g_4(\varphi_1\eta_2 - \varphi_3\eta_1) = 0 \\
\frac{\partial w_d}{\partial \eta_0} &= h_1(\varphi_1^2 + 2\varphi_2\varphi_3) + 2h_2\eta_1\eta_2 = 0
\end{aligned} \tag{4.2}$$

There are two un-equivalent solutions for the equations, one solution gives

$$\langle\varphi\rangle = v_\varphi\begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \langle\eta\rangle = v_\eta\begin{pmatrix}0\\1\end{pmatrix}, \quad \langle\chi\rangle = v_\chi\begin{pmatrix}0\\0\\1\end{pmatrix} \tag{4.3}$$

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with

$$v_\varphi = -\frac{g_2}{2g_1}v_\eta, \quad v_\chi = -\frac{g_2 g_4}{2g_1 g_3 M_\chi}v_\eta^2, \quad v_\eta \text{ undetermined} \tag{4.4}$$

another solution of the form $\langle v_\varphi\rangle = (1,1,1)^T v_\varphi, \langle\chi\rangle = (1,1,1)^T v_\chi, \langle\eta\rangle = (1,-1)^T v_\eta$ is forbidden by the last term in eq. (4.2).

The minimum equation of the two singlets σ and ϑ is simply given as

$$\frac{\partial w_d}{\partial \sigma_0} = q_1\sigma^2 + q_2\vartheta^2 = 0 \tag{4.5}$$

The equation in (4.5) lead to the following non-trivial solution

$$\langle\sigma\rangle = v_\sigma, \quad \langle\vartheta\rangle = v_\vartheta, \tag{4.6}$$

with

$$v_\sigma^2 = -\frac{q_2}{q_1}v_\vartheta^2, \quad v_\vartheta \text{ undetermined} \tag{4.7}$$

The VEVs of the above flavon fields in Φ^e will mainly determine the diagonal elements of mass matrices of charged fermions at LO, while the other two flavons ξ and ρ in Φ^e appear in off-diagonal entries at LO. Their vacuum configurations are easily obtained by the minimization equations

$$\begin{aligned}
\frac{\partial w_d}{\partial \xi_{01}} &= 2r_1(\xi_1^2 - \xi_2\xi_3) + r_2\xi_1\rho = 0\\
\frac{\partial w_d}{\partial \xi_{02}} &= 2r_1(\xi_2^2 - \xi_1\xi_3) + r_2\xi_3\rho = 0\\
\frac{\partial w_d}{\partial \xi_{03}} &= 2r_1(\xi_3^2 - \xi_1\xi_2) + r_2\xi_2\rho = 0
\end{aligned} \tag{4.8}$$

The equations in (4.8) lead to three sets of un-equivalent solutions. The first set is

$$\langle\xi\rangle = v_\xi \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\rho\rangle = v_\rho, \tag{4.9}$$

with

$$v_\xi = -\frac{r_2}{2r_1}v_\rho, \qquad v_\rho \text{ undetermined} \tag{4.10}$$

The second non-trivial solution is

$$\langle\xi\rangle = v_\xi \begin{pmatrix}0\\0\\0\end{pmatrix}, \quad \langle\rho\rangle = v_\rho, \tag{4.11}$$

with v_ρ undetermined. The third solution is

$$\langle\xi\rangle = v_\xi \begin{pmatrix}1\\1\\1\end{pmatrix}, \quad \langle\rho\rangle = 0, \tag{4.12}$$

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with v_ξ undetermined. The first solution in eq. (4.9) is chosen as the valid solution in our model. As elucidated in [68], the vacuum configurations of eqs. (4.3), (4.6), (4.9) and (4.14) are not the unique solutions of respective minimization equations. The other minima of the scalar potential can be derived by acting the original VEVs eqs. (4.3), (4.6), (4.9) and (4.14) with the elements of the flavor group S_4. Nevertheless, the new minima which are equivalent to the original ones can not lead to different physical results, i.e., the resulting fermion masses and mixing parameters are exactly the same as the original minima did. Hence without of generality the original vacuum alignments are chosed in the model, and the other scenarios with different phases only are related by the field redefinitions. Besides the non-trivial solutions, the trivial solutions with vanishing flavon VEVs cannot be excluded in principle, either. The problem could be solved by the introduction of the soft mass terms with the form $m_\xi^2|\xi|^2 + m_\rho^2|\rho|^2 + \tilde{m}_\xi^2\xi^2 + \tilde{m}_\rho^2\rho^2$ for ξ and ρ. By taking the mass parameters $m_{\xi,\rho}^2$ and $\tilde{m}_{\xi,\rho}^2$ to be negative values, one can check that only the configurations in eqs. (4.3), (4.6), (4.9) and (4.14) can be the lowest minimum of scalar potential and more stable than the vanishing ones.

The minimization equations for vacuum configurations of ϕ, Δ and ζ are given as following

$$\begin{aligned}
\frac{\partial w_d}{\partial \phi_{01}} &= f_1(\Delta_1\phi_2 - \Delta_2\phi_3) = 0 \\
\frac{\partial w_d}{\partial \phi_{02}} &= f_1(\Delta_1\phi_1 - \Delta_2\phi_2) = 0 \\
\frac{\partial w_d}{\partial \phi_{03}} &= f_1(\Delta_1\phi_3 - \Delta_2\phi_1) = 0 \\
\frac{\partial w_d}{\partial \Delta_{01}} &= f_2(\phi_3^2 + 2\phi_1\phi_2) + f_3\Delta_1^2 = 0 \\
\frac{\partial w_d}{\partial \Delta_{02}} &= f_2(\phi_2^2 + 2\phi_1\phi_3) + f_3\Delta_2^2 = 0 \\
\frac{\partial w_d}{\partial \zeta_0} &= f_4\zeta\zeta + f_5(\phi_1^2 + 2\phi_2\phi_3) + 2f_6\Delta_1\Delta_2 = 0
\end{aligned} \tag{4.13}$$

The solutions for above equations are listed as below

$$\langle\phi\rangle = v_\phi \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle\Delta\rangle = v_\Delta \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \langle\zeta\rangle = v_\zeta, \tag{4.14}$$

with the conditions

$$v_\phi^2 = -\frac{f_3}{3f_2}v_\Delta^2, \quad v_\Delta^2 = \frac{f_2 f_4}{f_3 f_5 - 2f_2 f_6}v_\zeta^2, \quad v_\zeta \text{ undetermined} \tag{4.15}$$

The solution (4.14) is used to produce the Tri-Bimaximal mixing pattern in the following sections.

The LO results in previous sections imply that the order of magnitude for $\langle\Phi^e\rangle$ and $\langle\Phi^\nu\rangle$ scaled by the cutoff Λ are not common. To be specific the order of VEVs are restricted as follows

$$\frac{v_\varphi}{\Lambda} \sim \frac{v_\eta}{\Lambda} \sim \frac{v_\chi}{\Lambda} \sim \frac{v_\sigma}{\Lambda} \sim \frac{v_\vartheta}{\Lambda} \sim \frac{v_\xi}{\Lambda} \sim \frac{v_\rho}{\Lambda} \sim \lambda_c^2, \qquad \frac{v_\phi}{\Lambda} \sim \frac{v_\Delta}{\Lambda} \sim \frac{v_\zeta}{\Lambda} \sim \lambda_c^3 \tag{4.16}$$

It is natural to require the subleading corrections to $\langle\Phi^e\rangle$ should be smaller than $m_\mu/m_\tau \sim \mathcal{O}(\lambda_c^2)$, or even more strictly smaller than $m_e/m_\tau \sim \mathcal{O}(\lambda_c^3)$. Because of the constrain of the auxiliary Abelian shaping symmetries $Z_4 \times Z_6 \times Z_5 \times Z_2$, the subleading corrections to superpotential w_d^e and w_d^ν are suppressed by $1/\Lambda$ and $1/\Lambda^2$, respectively, see appendix B for detail.

Next we shall briefly discuss the SU(5) GUT breaking scenario in the present scheme. The Higgs sector is composed of H_5, $H_{\bar{5}}$, H_{45}, $H_{\overline{45}}$, and the gauge group is broken by the VEV of the adjoint field H_{24}. The LO invariant interactions between the Higgs superfields and the adjoint field under the flavor symmetries are given by

$$\begin{aligned} w_H = & \frac{f_{H1}}{\Lambda} H_{24}H_{24}\sigma\rho + \sum_{i=1}^{5} \frac{f'_{Hi}}{\Lambda^3} H_{24}H_{24}\mathcal{O}_i^H + \frac{g_{H1}}{\Lambda^4} H_{\bar{5}}H_5H_{24}(\chi\chi)_2(\chi\phi)_2 \\ & + \frac{g_{H2}}{\Lambda^4} H_{\bar{5}}H_5H_{24}(\chi\chi)_{3_1}(\chi\phi)_{3_1} + \frac{g_{H3}}{\Lambda^4} H_{\bar{5}}H_5H_{24}(\chi\chi)_{3_1}(\chi\Delta)_{3_1}, \end{aligned} \tag{4.17}$$

in which the operators $\mathcal{O}_i^H$ include the following contractions of S_4

$$\mathcal{O}_i^H = \{\sigma\rho\zeta\zeta, \sigma\rho(\Delta\Delta)_{1_1}, \sigma\rho(\phi\phi)_{1_1}, \sigma\xi(\phi\phi)_{3_1}, \sigma\xi(\phi\Delta)_{3_1}\} \tag{4.18}$$

Plugging the VEVs of the flavons appear in the above equation, we can obtain the following

$$w_H = f_H H_{24}H_{24} + g_H H_{\bar{5}}H_5H_{24}, \tag{4.19}$$

with the coefficients

$$\begin{aligned} f_H &= \left(f_{H_1} + f'_{H_1}\frac{v_\zeta^2}{\Lambda^2} + 2f'_{H_2}\frac{v_\Delta^2}{\Lambda^2} + 3f'_{H_3}\frac{v_\phi^2}{\Lambda^2} \right)\frac{v_\sigma v_\rho}{\Lambda} + 2f'_{H_5}\frac{v_\sigma v_\xi v_\phi v_\Delta}{\Lambda^3}, \\ g_H &= (g_{H1} - 2g_{H2})\frac{v_\chi^3 v_\phi}{\Lambda^4} \end{aligned} \tag{4.20}$$

The superpotential in equation (4.17) breaks $\mathrm{U}(1)_R$ due to all the fields involved carrying 0 unit $\mathrm{U}(1)_R$ charge. Even taking into account the operators with the driving fields, Higgs fields and flavon fields combined together, the vanishing VEVs of the driving fields signify the operators have no contributions to the scalar potential. In order to completely understand the GUT symmetry breaking, one may consider the ultraviolet (UV) completion of the effective model. By adding the fields with non-zero unit $\mathrm{U}(1)_R$ charge, a $\mathrm{U}(1)_R$ conserving superpotential can give rise to the terms in eq. (4.17). For the examples of the ultraviolet completion in realistic flavor models, please see refs. [85, 86].

5 Corrections

The subleading corrections to superpotentials above arise from the higher dimensional operators which are suppressed by at least one power of $1/\Lambda$ and constrained by the symmetries. In this work the modified superpotential of driving fields w_d would give rise to the shifts of LO VEVs in Φ^e and Φ^ν. Fermions' correctional masses and mixing matrices are obtained by adding the higher order operators and the shifted vacua of flavon fields. The combinations $\phi\phi$, $\phi\Delta$, $\Delta\Delta$ and $\zeta\zeta$ are invariant under the auxiliary $\mathcal{G}_A = Z_4 \times Z_6 \times Z_5 \times Z_2$, hence it is always viable to add these combinations with any power on the top of each LO terms. It is expected that the subleading corrections are not impossible to destroy the LO predictions. Even though the expectation maybe correct, we would also like to present the detailed analysis of the subleading corrections.

5.1 Corrections to vacuum alignment

Here we just present the final results of shifted VEVs. The detailed calculation procedure is presented in appendix B. The modified VEVs of all flavons in Φ^e are of the form as follows

$$\langle\varphi\rangle = \begin{pmatrix} \delta v_{\varphi_1} \\ v_\varphi + \delta v_{\varphi_2} \\ \delta v_{\varphi_3} \end{pmatrix}, \quad \langle\eta\rangle = \begin{pmatrix} \delta v_{\eta_1} \\ v_\eta \end{pmatrix}, \quad \langle\chi\rangle = \begin{pmatrix} \delta v_{\chi_1} \\ \delta v_{\chi_2} \\ v_\chi + \delta v_{\chi_3} \end{pmatrix}, \quad \langle\xi\rangle = \begin{pmatrix} v_\xi + \delta v_{\xi_1} \\ \delta v_{\xi_2} \\ \delta v_{\xi_3} \end{pmatrix},$$
$$\langle\rho\rangle = v_\rho, \qquad \langle\sigma\rangle = v_\sigma + \delta v_\sigma, \quad \langle\vartheta\rangle = v_\vartheta, \tag{5.1}$$

with v_η, v_ϑ and $v_\rho$undetermined. We remark that the correctional results of each component of the flavons in Φ^e are different. Similarly the shifted vacua of the scalars in Φ^ν read

$$\langle\phi\rangle = \begin{pmatrix} v_\phi + \delta v_{\phi_1} \\ v_\phi + \delta v_{\phi_2} \\ v_\phi + \delta v_{\phi_3} \end{pmatrix}, \quad \langle\Delta\rangle = \begin{pmatrix} v_\Delta + \delta v_{\Delta_1} \\ v_\Delta + \delta v_{\Delta_2} \end{pmatrix}, \quad \langle\zeta\rangle = v_\zeta, \tag{5.2}$$

where v_ζ is still undetermined. We also remark that in fact the correctional results of each components of the flavons in Φ^ν are exactly the same, it is an important feature of the model which makes the Tri-Bimaximal mixing pattern still holds at subleading corrections in neutrino sector. Take into account the hierarchical VEVs $\langle\Phi^e\rangle/\Lambda \sim \lambda_c^2$ and $\langle\Phi^\nu\rangle/\Lambda \sim \lambda_c^3$, and the subleading operators linear in driving fields are suppressed by different power of $1/\Lambda$, the shifts also have different order of magnitude

$$\frac{\delta v_{\varphi_i}}{v_\varphi} \sim \frac{\delta v_{\eta_1}}{v_\eta} \sim \frac{\delta v_{\chi_i}}{v_\chi} \sim \lambda_c^4, \quad \frac{\delta v_\sigma}{v_\sigma} \sim \lambda_c^3, \quad \frac{\delta v_{\xi_{1,2}}}{v_\xi} = 0, \quad \frac{\delta v_{\xi_3}}{v_\xi} \sim \lambda_c^2, \quad \frac{\delta v_{\phi_i}}{v_\phi} \sim \frac{\delta v_{\Delta_i}}{v_\Delta} \sim \lambda_c^6 \tag{5.3}$$

5.2 Corrections to neutrino

Due to the auxiliary symmetry $Z_4 \times Z_6 \times Z_5 \times Z_2$, the subleading corrections to neutrino Majorana mass matrix only appear at next to next to leading order (NNLO). The higher dimensional operators arising from inserting bilinear invariant combinations of ϕ, Δ and ζ into the superfield N^cN^c in all possible ways, the corresponding terms are expressed explicitly as follows:

$$\frac{z_{c1}}{\Lambda}N^cN^c(\phi\phi)_{1_1} + \frac{z_{c2}}{\Lambda}N^cN^c(\phi\phi)_2 + \frac{z_{c3}}{\Lambda}N^cN^c(\phi\phi)_{3_1} + \frac{z_{c4}}{\Lambda}N^cN^c(\phi\Delta)_{3_1}$$
$$+\frac{z_{c5}}{\Lambda}N^cN^c(\Delta\Delta)_{1_1} + \frac{z_{c6}}{\Lambda}N^cN^c(\Delta\Delta)_2 + \frac{z_{c7}}{\Lambda}N^cN^c\zeta\zeta \tag{5.4}$$

Inserting the LO VEVs of the flavons in Φ^ν, the NNLO corrections to Majorana mass matrix will be

$$\delta M_M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{3z_{c1}v_\phi^2 + 2z_{c5}v_\Delta^2 + z_{c7}v_\zeta^2}{\Lambda} + \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \frac{3z_{c2}v_\phi^2 + z_{c6}v_\Delta^2}{\Lambda}$$
$$+ \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \frac{2z_{c4}v_\phi v_\Delta}{\Lambda} \tag{5.5}$$

We can note the form of δM_M is still compatible with TBM mixing. Denote the three terms that following the matrices as S_1, S_2, S_3 respectively, the eigenvalues of the subleading correctional Majorana masses are easily obtained as follows

$$dM_1 = S_1 - S_2 + 3S_3, \qquad dM_2 = S_1 + 2S_2, \qquad dM_3 = -S_1 + S_2 + 3S_3 \tag{5.6}$$

The possible content that spoils the TBM mixing only arising from Dirac mass terms, whose corresponding subleading superpotential is comprised of the shifted VEVs at LO and higher dimensional operators as following

$$\delta w_D = \sum_{i=1}^{2} \frac{y_{\nu_i}}{\Lambda} F N^c \delta\Phi_i^\nu H_5 + \sum_{i=1}^{12} \frac{y'_{\nu_i}}{\Lambda^3} (FN^c)_{c_i} (\Phi^\nu\Phi^\nu\Phi^\nu)_{c_i} H_5 \tag{5.7}$$

in which the $\delta\Phi_i^\nu$ denotes the shifted vacua of flavon Φ_i^ν, and c_i indicate all possible S_4 contractions. Substituting the unique LO vacua structures of Φ^ν in eq. (4.14) and the shifted VEVs of $\delta\Phi^\nu$ in eq. (B.24) into eq. (5.7), one can check that actually the two sets of superpotential invariants still maintain TBM mixing after symmetry breaking, and the modified Dirac mass terms are exactly the same as the mass structure in eq. (3.9) at LO and in eq. (5.5) at NNLO, thus the corrections could be absorbed into Yukawa couplings by redefining parameters y_{ν_i} in the LO part. Note that the contraction 1_1 of S_4 also induces extra term in Dirac mass matrix, which arise from the second term in above eq. (5.7), i.e., the terms with $c_i = 1_1$, however, can not be absorbed into the redefinition of parameters y_{ν_i}. To be specific the correctional extra Dirac mass matrix is found to be

$$\delta m_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{v_5}{\Lambda} c, \quad c = (6y'_{\nu_1} v_\phi^2 v_\Delta + 2y'_{\nu_2} v_\Delta^3)/\Lambda^2. \tag{5.8}$$

Collected all the Dirac mass matrices in eq. (3.9) and eq. (5.8), the structure is exact the same with eq. (5.5), and can be still diagonalized by TBM mixing matrix. The stability of TBM mixing in neutrino sector is guaranteed by the stable VEV structures of Φ^ν, see appendix B. It is a salient feature of the model that Tri-Bimaximal mixing still holds even at NNLO.

Take into account these subleading contributions to both Dirac and Majorana mass matrices, we can easily rewrite the modified light neutrino masses as follows

$$m_1 = \left| -\frac{(3a - b + c)^2}{M + dM_1} \right| \frac{v_5^2}{\Lambda^2}, \quad m_2 = \left| -\frac{(2b + c)^2}{M + dM_2} \right| \frac{v_5^2}{\Lambda^2}, \quad m_3 = \left| \frac{(3a + b - c)^2}{M - dM_3} \right| \frac{v_5^2}{\Lambda^2} \tag{5.9}$$

Note that a, b in above expressions are not exactly the same as in eq. (3.13), because of the redefinition of LO parameters y_{ν_i} by absorbing the corrections to Dirac masses in eq. (5.7). However the subleading corrections are too small to change the order of magnitudes, we can treat them unchanged. The extra term c is also extremely small compared with a and b, which can not affect the magnitude of neutrino mass as well.

5.3 Corrections to charged fermions

There are two sources for the correctional contributions to charged fermions, one is the higher-dimensional operators, another is the shifted VEVs at LO. The masses and mixings for charged fermions have been well determined at LO, hence the subleading effects are expected to be negligible. For sake of simplicity, we shall drop the detailed procedure of calculations, and focus on the generical results of subleading effects. Ignoring the $\mathcal{O}(1)$ couplings and setting all cutoff to be Λ, the higher dimensional invariant operators under all the symmetries are generically of the form

$$\begin{aligned}
\delta w_U &\supset T_i T_j \left(\frac{\Phi^e}{\Lambda}\right)^m \left(\frac{\Phi^\nu}{\Lambda}\right)^n H_{5,45}, \\
\delta w_D &\supset T_i F \left(\frac{\Phi^e}{\Lambda}\right)^p \left(\frac{\Phi^\nu}{\Lambda}\right)^q H_{\bar{5},\overline{45}} + T_i F H_{24} H_{\bar{5},\overline{45}} \left(\frac{\Phi^e}{\Lambda}\right)^r \left(\frac{\Phi^\nu}{\Lambda}\right)^s
\end{aligned} \tag{5.10}$$

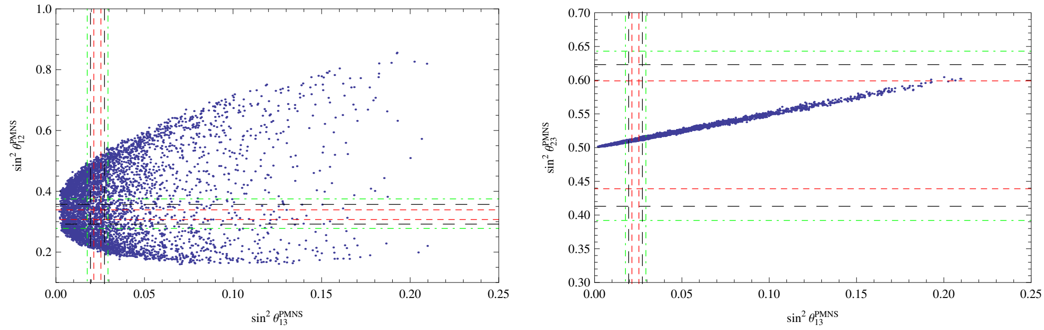
with $m+n \geqslant 4$, $p+q \geqslant 4$ and $r+s \geqslant 4$. The order of $\langle\Phi^e\rangle$ and $\langle\Phi^\nu\rangle$ imply that the subleading contributions to the entries of M_U and M_D from δw_U and the first term of δw_D are of order ϵ^4 or less. Depending on the ways SU(5) indices contract, the largest corrections of order $\epsilon^2\delta$ to second column of M_D arise from the operators $[FH_{\bar{5}}]_{\mathbf{15}}[T_2 H_{24}]_{\overline{\mathbf{15}}}\sigma(\xi\phi,\xi\Delta)_{3_1}$. Also the operators give rise to vanishing contributions to M_ℓ due to $y_e/y_d = 0$ [99]. In short the higher-dimensional operators contribute negligible effect with respect to the LO results in section 3.

Plugging the shifted vacua δv_Φ in eq. (5.3) into the LO superpotential in section 3, one may check that the correctional mass entries are much smaller than respective LO ones. All the corrections have no significant impact on the LO masses, $m_{u,c,t}$, $m_{d,s,b}$ and $m_{e,\mu,\tau}$, and mixings $V_{\rm CKM}$ and $U_{\rm PMNS}$. In conclusion the LO predictions can not be spoiled by the two types of subleading effects.

6 Phenomenology

The model we have constructed has only analytic form despite of some parameters within it. In the next step we shall present the phenomenological numerical results of some observables we interested. We require the input oscillation parameters $\Delta m^2_{\rm sol}$, $\Delta m^2_{\rm atm}$, $\sin^2\theta_{12}$, $\sin^2\theta_{13}$, and $\sin^2\theta_{23}$ to lie in their 3σ intervals which are taken from ref. [107]. It is easy to express the light neutrino mass spectrum with well determined mass differences and the lightest neutrino mass $m_{\rm lightest} = m_1$ (m_3) in NH (IH) spectrum as follows

$$\begin{aligned}
&\text{NH}: m_1 < m_2 < m_3, \quad m_2 = \sqrt{\Delta m^2_{\rm sol} + m_1^2}, \quad m_3 = \sqrt{\Delta m^2_{\rm atm} + m_1^2} \\
&\text{IH}: \ m_3 < m_1 < m_2, \quad m_1 = \sqrt{m_3^2 - \Delta m^2_{\rm atm}}, \quad m_2 = \sqrt{m_3^2 - \Delta m^2_{\rm atm} + \Delta m^2_{\rm sol}}
\end{aligned} \tag{6.1}$$



(a) (b)

Figure 1. The allowed region of $\sin^2\theta_{12}^{\rm PMNS} - \sin^2\theta_{13}^{\rm PMNS}$ (left panel) and $\sin^2\theta_{23}^{\rm PMNS} - \sin^2\theta_{13}^{\rm PMNS}$ (right panel).

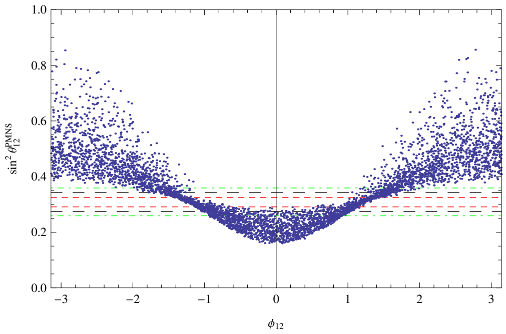
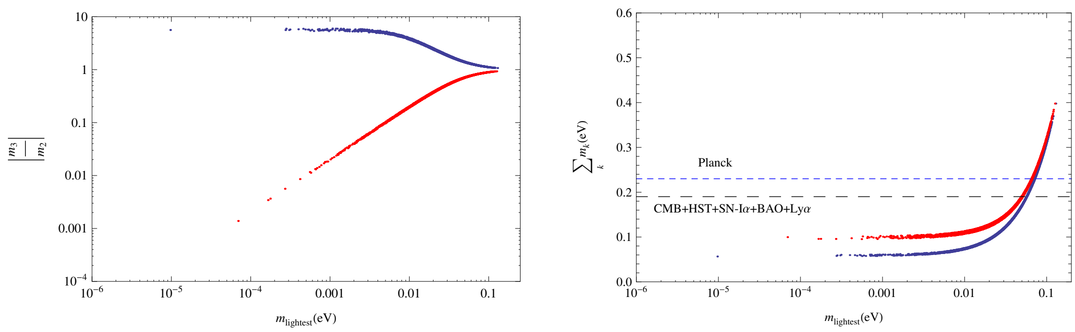


Figure 2. Correlation of $\sin^2\theta_{12}^{\rm PMNS}$ and $\phi_{12} = \arg(y_{12}^d/y_{22}^d)$.

6.1 Mixing angles

In the numerical analysis all the coupling coefficients in analytic expressions, i.e., eq. (3.52), of mixing angles are taken to random complex numbers with their absolute values (or say modulus) within an interval $[1/2,\, 3/2]$, and the small parameters ϵ and δ can be fixed at 0.05 and 0.01 respectively as demonstration values. The ratio $v_{\overline{45}}/v_{\bar{5}}$ is chosen the typical value 0.22. The analytic expressions of leptonic mixing angles are shown in eq. (3.52), which include the deviations from TBM mixing values. Thus we can estimate the allowed region of mixing parameters numerically. The allowed regions of $\sin^2\theta_{13}^{\rm PMNS} - \sin^2\theta_{12}^{\rm PMNS}$ and $\sin^2\theta_{13}^{\rm PMNS} - \sin^2\theta_{23}^{\rm PMNS}$ are shown in figure 1(a) and figure 1(b) respectively. The horizontal lines show the 3σ (green), 2σ (black) and 1σ (red) boundaries of the mixing angles $\theta_{12}^{\rm PMNS}$ and $\theta_{23}^{\rm PMNS}$, while the vertical line presents the corresponding boundaries of $\theta_{13}^{\rm PMNS}$.

As showed in eq. (3.54), the deviation of $\theta_{12}^{\rm PMNS}$ from its TBM value θ_{12}^{ν} is mainly controlled by the complex phase, defined as ϕ_{12}, of y_{12}^d/y_{22}^d in which we have restricted the module of y_{12}^d/y_{22}^d in the interval $[1/3,\, 3]$. Thus we can estimate the effect of the phase on mixing angle $\theta_{12}^{\rm PMNS}$, see figure 2. The horizontal lines stand for the confidence level as in figure 1(a), and it is obviously that $\phi_{12} \sim \pi/2$ or $-\pi/2$ are favoured so that $\theta_{12}^{\rm PMNS}$ can lie in the 3σ interval in the present model.



(a) (b)

Figure 3. Scatter plots of $|m_3/m_2|$ (left panel) and sum of neutrino masses $\sum_k m_k$ (right panel) as the function of the lightest neutrino mass m_{lightest}. In both plots blue corresponds to NH mass spectrum and red to IH mass spectrum.

6.2 Sum of neutrino masses, Neutrinoless double beta decay

First we consider two simple arithmetical relationship between light neutrino masses: the ratio $|m_3/m_2|$ and sum of all masses against the lightest neutrino mass. The plot of the ratio $|m_3/m_2|$ and sum of light neutrino mass $\sum_k m_k$ as function of the lightest neutrino mass m_{lightest}, which is m_1 (m_3) for NH (IH) mass spectrum, are shown in figure 3(a) and figure 3(b), respectively. Note that the horizontal lines in figure 3(b) represents the cosmological bound at 0.19 eV (black), corresponding to the combined observational data from [113–122], and the upper bounds 0.23 eV from Planck [123]. The ratio tends to a degenerate mass spectrum in both cases as the value of $m_{\text{lightest}} \to 0.1$eV which is disfavoured in the model. The masses sum $\sum_k m_k$ in the model is predicted too similar for both hierarchical mass spectrums to be distinguished using the cosmological bound on the sum of neutrino masses.

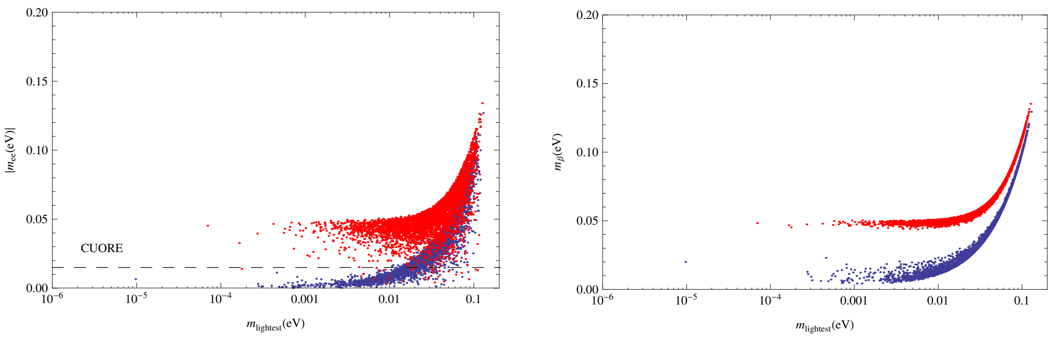
The effective Majorana mass $|m_{ee}|$ determines the $0\nu\beta\beta$ decay amplitude, which is also the (11) element of neutrino mass matrix in flavor basis, and meant a real and diagonal matrix of charged lepton mass. It is defined as follows

$$|m_{ee}| = \left|\sum_i (U_{\text{PMNS}})^2_{ei} m_i\right| \tag{6.2}$$

and the β decay effective mass which could measure the non-zero neutrino masses is

$$m_\beta = \left[\sum_k \left|(U_{\text{PMNS}})_{ek}\right|^2 m_k^2\right]^{1/2} \tag{6.3}$$

In the numerical analysis the mass differences are well-known input parameters, as explained in the beginning of this section, and the analytic form of m_{lightest} (m_1 or m_3) in eq. (5.9) are the single variant of physical quantities $|m_{ee}|$ and m_β. The predicted allowed region of the two effective masses against lightest mass, are shown in figure 4(a) and figure 4(b). The dashed line in figure 4(a) shows the future sensitivity of CUORE [124] experiment at 15 meV. In the model $|m_{ee}|$ is predicted to below the upper limit of $|m_{ee}|$,



(a) (b)

Figure 4. Effective Majorana mass of $0\nu\beta\beta$ decay $|m_{ee}|$ (left panel) and effective mass of beta decay (right panel) m_β as the function of the lightest neutrino mass m_{lightest}. In both plots blue corresponds to NH mass spectrum and red to IH mass spectrum.

which is constrained from the Heidelberg-Moscow experiment [125]. As the m_{lightest} grows to be around 0.1 eV, $|m_{ee}|$ tends to degenerate in both NH and IH mass spectrum.

The β decay effective mass m_β is predicted to be below the future sensitivity 0.2eV of the KATRIN [126] experiment, as showed in figure 4(b). The vertical line represents the sensitivity.

7 Conclusion

In this paper, we have proposed a flavor model in the framework of SUSY SU(5) GUT based on $S_4 \times Z_4 \times Z_6 \times Z_5 \times Z_2$ flavor symmetry. In the model the first and third generations of **10** dimensional representation in SU(5) are all assigned to be 1_1 of S_4. The second generation of **10** dimensional is to be 1_2 of S_4. Right-handed neutrinos of singlet **1** in SU(5) and three generations of $\bar{\mathbf{5}}$ in SU(5) are all assigned to be 3_1 of S_4. The flavons in Φ are divided into Φ^e in charged fermion sector and Φ^ν in neutrino sector, whose VEVs are of different orders of magnitude: $\langle\Phi^e\rangle/\Lambda \sim \lambda_c^2$ and $\langle\Phi^\nu\rangle/\Lambda \sim \lambda_c^3$, with λ_c being Cabibbo angle. Also the energy scale Λ is below the GUT scale.

The three right-handed neutrinos are SU(5) singlets, and the light neutrino masses are generated via type-I seesaw mechanism only. The diagonalization of neutrino mass matrix leads to Tri-Bimaximal mixing pattern at LO, both normal and inverted hierarchy mass spectrums are allowed. The subleading corrections to both Majorana and Dirac masses arise from the higher dimensional operators and shifted VEVs lead to the same mass structures as that at LO, and give no change to the TBM mixing pattern. It is a salient feature of the present model that TBM mixing still holds exactly at NNLO.

The mass hierarchies of up-type quarks are controlled by the spontaneously symmetry broken. The top quark obtains its mass purely at tree level, the LO mass of charm quark is derived only by two flavons σ and ϑ in Φ^e, while the LO mass of up quark is controlled by three flavons: η in Φ^e and Δ, ζ in Φ^ν. Due to the moderate hierarchy assumptions $\langle\Phi^e\rangle \sim \lambda_c^2\Lambda$ and $\langle\Phi^\nu\rangle \sim \lambda_c^3\Lambda$, the phenomenological favored mass hierarchies of all three generations of up-type quarks are obtained, i.e., $m_u : m_c : m_t = \lambda_c^8 : \lambda_c^4 : 1$. The mixing at LO only exists between the last two generations, give rise to the mixing angle $\theta_{23}^u \sim \lambda_c^2$.

The mass texture of down-type quarks is similar to that of charged-leptons due to the same set of GUT operators, despite of the different CG factors and transposed relations. The model predicts that bottom-tau unification $m_b = m_\tau$ as well as the popular Georgi-Jarlskog relation $m_\mu = 3m_s$. In addition the model also gives a new mass relation between electron and down quark, namely $m_e = \frac{8}{27} m_d$. The new ratio arises from the splitted masses of heavy messenger fields by a specific novel CG factors from GUT symmetry breaking. Concretely the non-singlet field is an adjoint representation, H_{24}, of SU(5) in our model. The novel CG factors caused by $\langle H_{24} \rangle$ enter inversely in the desired Yukawa textures of quark-lepton, leading to the new mass relation. The model also gives the CKM quark mixing matrix and a CKM-like mixing matrix of charged leptons. The resulting Cabibbo angle θ_c between the first two families of down-quarks, together with the mixing angle between the first and third generations all require a fine tuning $v_{\overline{45}}/v_{\bar{5}} \sim \lambda_c$. Combined the mixing angle θ^u_{23}, all the elements of CKM matrix can be derived properly. On the other hand the CKM-like mixing matrix of charged leptons also implies a sizable mixing between electron and muon, given that $\theta^e_{12} \simeq \lambda_c$. The mixing angle is constrained by the GUT relation $\theta^e_{12} = |\frac{c_c}{c_b}| \frac{m_e}{m_\mu} \frac{1}{\theta^d_{12}}$, only with the condition that $|\frac{c_c}{c_b}| \sim \mathcal{O}(2/\lambda_c)$ and $\theta^d_{12} \simeq \lambda_c$, the angle θ^e_{12} can be λ_c as well. The model with the novel CG factors indeed leads to $\frac{c_c}{c_b} = \frac{81}{8}$, which satisfies the condition. Finally the CKM-like mixing matrix of charged leptons with $\theta^e_{12} \simeq \lambda_c$ modifies the vanishing θ^ν_{13} in TBM mixing pattern to a sizable lepton mixing angle $\theta^{\mathrm{PMNS}}_{13} \simeq \lambda_c/\sqrt{2}$, in well compatible with experimental results.

We also present the subleading corrections to flavon alignment in detail, and we find that all VEVs $\langle \Phi^\nu \rangle$ actually receive very small shifts along the same directions of the LO alignment even at NNLO corrections, but it is not the case for Φ^e. The stable solutions of $\langle \Phi^\nu \rangle$ also guarantee the stability of TBM mixing in neutrino sector. The subleading corrections to charged fermions are negligible with respect to the LO predictions. The final results are not altered in order of magnitude even in the expressions. In the end we show the phenomenological numerical results predicted by the model. Future long base line neutrino experiments with higher precision is possible to verify or falsify the result about leptonic CP violating phase predicted in the model. The neutrinoless double beta decay experiment is also a tool for testing the model, which can discriminate between the NH spectrum and the IH one.

Acknowledgments

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A S_4 group and representations

The discrete flavor group S_4, permutation group of four objects, has 24 elements. The two generators S and T in different irreducible presentations are given as follows

$$1_1: \quad S = 1, \qquad T = 1 \tag{A.1}$$

$$1_2: \quad S = -1, \qquad T = 1 \tag{A.2}$$

	1_1	1_2	2	3_1	3_2
$\mathcal{C}_1$	1	1	2	3	3
$\mathcal{C}_2$	1	1	2	-1	-1
$\mathcal{C}_3$	1	1	-1	0	0
$\mathcal{C}_4$	1	-1	0	1	-1
$\mathcal{C}_5$	1	-1	0	-1	1

Table 3. The character table of S_4 group.

$$2: \quad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} \tag{A.3}$$

$$3_1: \quad S = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \tag{A.4}$$

$$3_2: \quad S = \frac{1}{3}\begin{pmatrix} 1 & -2\omega & -2\omega^2 \\ -2\omega & -2\omega^2 & 1 \\ -2\omega^2 & 1 & -2\omega \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \tag{A.5}$$

where $\omega = e^{2\pi i/3} = (i\sqrt{3}-1)/2$. The five conjugate classes of S_4 group can be written as

$$\begin{array}{lllllllll}
\mathcal{C}_1: & 1 & & & & & & & \\
\mathcal{C}_2: & S^2, & TS^2T^2, & T^2S^2T & & & & & \\
\mathcal{C}_3: & T, & T^2, & S^2T, & S^2T^2, & STST^2, & STS, & TS^2, & T^2S^2 \\
\mathcal{C}_4: & ST^2, & T^2S, & TST, & TSTS^2, & STS^2, & S^2TS & & \\
\mathcal{C}_5: & S, & TST^2, & ST, & TS, & S^3, & S^3T^2 & &
\end{array} \tag{A.6}$$

The character table of S_4 group are shown in table 3 and the multiplication rules between various irreducible representations are shown in eq. (3). Taking into account of the generators S and T in different representations, one may obtain the representation matrices of all elements. The explicit expressions of S_4 elements in different representations can be found in refs. [127, 128], especially the subgroups of S_4 are thorough classified in [128]. In this basis we can straightforwardly obtain the decomposition of the product representations and the Clebsch-Gordan factors. To be specific the product rules of S_4 group, with ψ_i, φ_i as the elements of the first and second representation of the product, respectively, are given as follows

$$\star \quad 1_1 \otimes r = r \otimes 1_1 = r \qquad \text{with } r \text{ being any representation} \tag{A.7}$$

$$\star \quad 1_2 \otimes 1_2 = 1_1 \sim \psi\varphi \tag{A.8}$$

$$\star \quad 1_2 \otimes 2 = 2 \sim \begin{pmatrix} \psi\varphi_1 \\ -\psi\varphi_2 \end{pmatrix} \tag{A.9}$$

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$$\star \quad 1_2 \otimes 3_1 = 3_2 \sim \begin{pmatrix} \psi\varphi_1 \\ \psi\varphi_2 \\ \psi\varphi_3 \end{pmatrix} \tag{A.10}$$

$$\star \quad 1_2 \otimes 3_2 = 3_1 \sim \begin{pmatrix} \psi\varphi_1 \\ \psi\varphi_2 \\ \psi\varphi_3 \end{pmatrix} \tag{A.11}$$

The product rules with two-dimensional representation are as follows:

$$\begin{aligned} \star \quad & 2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2 \\ & 1_1 \sim \psi_1\varphi_2 + \psi_2\varphi_1, \qquad 1_2 \sim \psi_1\varphi_2 - \psi_2\varphi_1 \\ & 2 \sim \begin{pmatrix} \psi_2\varphi_2 \\ \psi_1\varphi_1 \end{pmatrix} \end{aligned} \tag{A.12}$$

$$\begin{aligned} \star \quad & 2 \otimes 3_1 = 3_1 \oplus 3_2 \\ & 3_1 \sim \begin{pmatrix} \psi_1\varphi_2 + \psi_2\varphi_3 \\ \psi_1\varphi_3 + \psi_2\varphi_1 \\ \psi_1\varphi_1 + \psi_2\varphi_2 \end{pmatrix}, \qquad 3_2 \sim \begin{pmatrix} \psi_1\varphi_2 - \psi_2\varphi_3 \\ \psi_1\varphi_3 - \psi_2\varphi_1 \\ \psi_1\varphi_1 - \psi_2\varphi_2 \end{pmatrix} \end{aligned} \tag{A.13}$$

$$\begin{aligned} \star \quad & 2 \otimes 3_2 = 3_1 \oplus 3_2 \\ & 3_1 \sim \begin{pmatrix} \psi_1\varphi_2 - \psi_2\varphi_3 \\ \psi_1\varphi_3 - \psi_2\varphi_1 \\ \psi_1\varphi_1 - \psi_2\varphi_2 \end{pmatrix}, \qquad 3_2 \sim \begin{pmatrix} \psi_1\varphi_2 + \psi_2\varphi_3 \\ \psi_1\varphi_3 + \psi_2\varphi_1 \\ \psi_1\varphi_1 + \psi_2\varphi_2 \end{pmatrix} \end{aligned} \tag{A.14}$$

and the product rules with three-dimensional representation are as follows

$$\begin{aligned} \star \quad & 3_1 \otimes 3_1 = 3_2 \otimes 3_2 = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2 \\ & 1_1 \sim \psi_1\varphi_1 + \psi_2\varphi_3 + \psi_3\varphi_2 \\ & 2 \sim \begin{pmatrix} \psi_2\varphi_2 + \psi_3\varphi_1 + \psi_1\varphi_3 \\ \psi_3\varphi_3 + \psi_1\varphi_2 + \psi_2\varphi_1 \end{pmatrix} \\ & 3_1 \sim \begin{pmatrix} 2\psi_1\varphi_1 - \psi_2\varphi_3 - \psi_3\varphi_2 \\ 2\psi_3\varphi_3 - \psi_1\varphi_2 - \psi_2\varphi_1 \\ 2\psi_2\varphi_2 - \psi_3\varphi_1 - \psi_1\varphi_3 \end{pmatrix}, \qquad 3_2 \sim \begin{pmatrix} \psi_2\varphi_3 - \psi_3\varphi_2 \\ \psi_1\varphi_2 - \psi_2\varphi_1 \\ \psi_3\varphi_1 - \psi_1\varphi_3 \end{pmatrix} \end{aligned} \tag{A.15}$$

$$\begin{aligned} \star \quad & 3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_1 \oplus 3_2 \\ & 1_2 \sim \psi_1\varphi_1 + \psi_2\varphi_3 + \psi_3\varphi_2 \\ & 2 \sim \begin{pmatrix} \psi_2\varphi_2 + \psi_3\varphi_1 + \psi_1\varphi_3 \\ -\psi_3\varphi_3 - \psi_1\varphi_2 - \psi_2\varphi_1 \end{pmatrix} \\ & 3_1 \sim \begin{pmatrix} \psi_2\varphi_3 - \psi_3\varphi_2 \\ \psi_1\varphi_2 - \psi_2\varphi_1 \\ \psi_3\varphi_1 - \psi_1\varphi_3 \end{pmatrix}, \qquad 3_2 \sim \begin{pmatrix} 2\psi_1\varphi_1 - \psi_2\varphi_3 - \psi_3\varphi_2 \\ 2\psi_3\varphi_3 - \psi_1\varphi_2 - \psi_2\varphi_1 \\ 2\psi_2\varphi_2 - \psi_3\varphi_1 - \psi_1\varphi_3 \end{pmatrix} \end{aligned} \tag{A.16}$$

B Corrections to vacuum alignment

The subleading corrections to LO vacuum alignment stem from higher dimensional operators which are suppressed by one or more power of $1/\Lambda$. The modified driving superpotential will consist of leading order term w_d^0, which is just eq. (4.1), and correctional term δw_d, which comes from all invariant operators linear in the driving fields that suppressed by $1/\Lambda$ at least one power

$$w_d = w_d^0 + \delta w_d \tag{B.1}$$

The correctional term δw_d contains all contractional invariant subdominant operators under the flavor symmetry $S_4 \times Z_4 \times Z_6 \times Z_5 \times Z_2$

$$\begin{aligned}\delta w_d = &\sum_{i=1}^{5} \frac{a_i}{\Lambda}\mathcal{O}_i^{\varphi_0} + \sum_{i=1}^{8} \frac{b_i}{\Lambda}\mathcal{O}_i^{\chi_0} + \frac{c}{\Lambda}\mathcal{O}^{\eta_0} + \sum_{i=1}^{3} \frac{m_i}{\Lambda}\mathcal{O}^{\sigma_0} + \frac{k}{\Lambda}\mathcal{O}^{\xi_0} \\ &+ \sum_{i=1}^{12} \frac{s_i}{\Lambda^2}\mathcal{O}_i^{\phi_0} + \sum_{i=1}^{12} \frac{t_i}{\Lambda^2}\mathcal{O}_i^{\Delta_0} + \sum_{i=1}^{11} \frac{u_i}{\Lambda^2}\mathcal{O}_i^{\zeta_0}\end{aligned} \tag{B.2}$$

in which the complex coefficients a_i, b_i, c, d_i, m, k, s_i, t_i and u_i are all of order one but cannot be specific determined according to the flavor symmetry. Operators $\mathcal{O}_i$ represent all the subdominant invariant contractional operators under the symmetry group $S_4 \times Z_4 \times Z_6 \times Z_5 \times Z_2$

$$\begin{aligned}&\mathcal{O}_1^{\varphi_0} = (\varphi_0\chi)_2(\phi\phi)_2, && \mathcal{O}_2^{\varphi_0} = (\varphi_0\chi)_{3_1}(\phi\phi)_{3_1}, && \mathcal{O}_3^{\varphi_0} = (\varphi_0\chi)_{3_1}(\phi\Delta)_{3_1}\\ &\mathcal{O}_4^{\varphi_0} = (\varphi_0\chi)_{3_2}(\phi\Delta)_{3_2}, && \mathcal{O}_5^{\varphi_0} = (\varphi_0\chi)_2(\Delta\Delta)_2, &&\end{aligned} \tag{B.3}$$

$$\begin{aligned}&\mathcal{O}_1^{\chi_0} = (\chi_0\chi)_{1_1}(\phi\phi)_{1_1}, && \mathcal{O}_2^{\chi_0} = (\chi_0\chi)_2(\phi\phi)_2, && \mathcal{O}_3^{\chi_0} = (\chi_0\chi)_{3_1}(\phi\phi)_{3_1}\\ &\mathcal{O}_4^{\chi_0} = (\chi_0\chi)_{3_1}(\phi\Delta)_{3_1}, && \mathcal{O}_5^{\chi_0} = (\chi_0\chi)_{3_2}(\phi\Delta)_{3_2} && \\ &\mathcal{O}_6^{\chi_0} = (\chi_0\chi)_{1_1}(\Delta\Delta)_{1_1}, && \mathcal{O}_7^{\chi_0} = (\chi_0\chi)_2(\Delta\Delta)_2, && \mathcal{O}_8^{\chi_0} = (\chi_0\chi)_{1_1}\zeta\zeta\end{aligned} \tag{B.4}$$

$$\mathcal{O}^{\eta_0} = \eta_0\chi(\phi\Delta)_{3_2} \tag{B.5}$$

$$\mathcal{O}_1^{\sigma_0} = \sigma_0(\varphi\chi)_{3_1}\phi, \qquad \mathcal{O}_2^{\sigma_0} = \sigma_0(\varphi\chi)_2\Delta, \qquad \mathcal{O}_3^{\sigma_0} = \sigma_0(\eta\chi)_{3_1}\phi \tag{B.6}$$

$$\mathcal{O}^{\xi_0} = \xi_0(\xi\chi)_{3_1}\vartheta, \tag{B.7}$$

$$\begin{aligned}&\mathcal{O}_1^{\phi_0} = \phi_0(\phi\Delta)_{3_2}\zeta\zeta, && \mathcal{O}_2^{\phi_0} = \phi_0\big((\phi\phi)_2(\phi\phi)_{3_1}\big)_{3_2}, && \mathcal{O}_3^{\phi_0} = \phi_0\big((\phi\phi)_{3_1}(\phi\phi)_{3_1}\big)_{3_2},\\ &\mathcal{O}_4^{\phi_0} = \phi_0(\phi\phi)_{1_1}(\phi\Delta)_{3_2}, && \mathcal{O}_5^{\phi_0} = \phi_0\big((\phi\phi)_2(\phi\Delta)_{3_1}\big)_{3_2}, && \mathcal{O}_6^{\phi_0} = \phi_0\big((\phi\phi)_2(\phi\Delta)_{3_2}\big)_{3_2},\\ &\mathcal{O}_7^{\phi_0} = \phi_0\big((\phi\phi)_{3_1}(\phi\Delta)_{3_1}\big)_{3_2}, && \mathcal{O}_8^{\phi_0} = \phi_0\big((\phi\phi)_{3_1}(\phi\Delta)_{3_2}\big)_{3_2}, && \mathcal{O}_9^{\phi_0} = \phi_0\big((\phi\phi)_{3_1}(\Delta\Delta)_2\big)_{3_2},\\ &\mathcal{O}_{10}^{\phi_0} = \phi_0(\phi\Delta)_{3_2}(\Delta\Delta)_{1_1}, && \mathcal{O}_{11}^{\phi_0} = \phi_0\big((\phi\Delta)_{3_1}(\Delta\Delta)_2\big)_{3_2}, && \mathcal{O}_{12}^{\phi_0} = \phi_0\big((\phi\Delta)_{3_2}(\Delta\Delta)_2\big)_{3_2}\end{aligned} \tag{B.8}$$

$$\begin{aligned}&\mathcal{O}_1^{\Delta_0} = \Delta_0(\phi\phi)_{1_1}(\phi\phi)_2, && \mathcal{O}_2^{\Delta_0} = \Delta_0\big((\phi\phi)_2(\phi\phi)_2\big)_2, && \mathcal{O}_3^{\Delta_0} = \Delta_0\big((\phi\phi)_{3_1}(\phi\phi)_{3_1}\big)_2,\\ &\mathcal{O}_4^{\Delta_0} = \Delta_0\big((\phi\phi)_{3_1}(\phi\Delta)_{3_1}\big)_2, && \mathcal{O}_5^{\Delta_0} = \Delta_0\big((\phi\phi)_{3_1}(\phi\Delta)_{3_2}\big)_2, && \mathcal{O}_6^{\Delta_0} = \Delta_0(\phi\phi)_{1_1}(\Delta\Delta)_2,\\ &\mathcal{O}_7^{\Delta_0} = \Delta_0(\phi\phi)_2(\Delta\Delta)_{1_1}, && \mathcal{O}_8^{\Delta_0} = \Delta_0\big((\phi\phi)_2(\Delta\Delta)_2\big)_2, && \mathcal{O}_9^{\Delta_0} = \Delta_0(\phi\phi)_2\zeta\zeta,\\ &\mathcal{O}_{10}^{\Delta_0} = \Delta_0(\Delta\Delta)_{1_1}(\Delta\Delta)_2, && \mathcal{O}_{11}^{\Delta_0} = \Delta_0(\Delta\Delta)_2(\Delta\Delta)_2, && \mathcal{O}_{12}^{\Delta_0} = \Delta_0(\Delta\Delta)_2\zeta\zeta,\end{aligned} \tag{B.9}$$

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$$
\begin{aligned}
&\mathcal{O}_1^{\zeta_0}=\zeta_0\zeta\zeta(\phi\phi)_{1_1}, && \mathcal{O}_2^{\zeta_0}=\zeta_0\zeta\zeta(\Delta\Delta)_{1_1}, && \mathcal{O}_3^{\zeta_0}=\zeta_0\zeta\zeta\zeta\zeta,\\
&\mathcal{O}_4^{\zeta_0}=\zeta_0(\phi\phi)_{1_1}(\phi\phi)_{1_1}, && \mathcal{O}_5^{\zeta_0}=\zeta_0\big((\phi\phi)_2(\phi\phi)_2\big)_{1_1}, && \mathcal{O}_6^{\zeta_0}=\zeta_0\big((\phi\phi)_{3_1}(\phi\phi)_{3_1}\big)_{1_1},\\
&\mathcal{O}_7^{\zeta_0}=\zeta_0\big((\phi\phi)_{3_1}(\phi\Delta)_{3_1}\big)_{1_1}, && \mathcal{O}_8^{\zeta_0}=\zeta_0(\phi\phi)_{1_1}(\Delta\Delta)_{1_1}, && \mathcal{O}_9^{\zeta_0}=\zeta_0\big((\phi\phi)_2(\Delta\Delta)_2\big)_{1_1},\\
&\mathcal{O}_{10}^{\zeta_0}=\zeta_0(\Delta\Delta)_{1_1}(\Delta\Delta)_{1_1}, && \mathcal{O}_{11}^{\zeta_0}=\zeta_0\big((\Delta\Delta)_2(\Delta\Delta)_2\big)_{1_1}
\end{aligned}
\tag{B.10}
$$

The sub-dominate term δw_d induces shifted VEVs of all flavon fields, and we can rewrite the modified vacuum alignment as following

$$
\begin{aligned}
&\langle\varphi\rangle=\begin{pmatrix}\delta v_{\varphi_1}\\ v_\varphi+\delta v_{\varphi_2}\\ \delta v_{\varphi_3}\end{pmatrix}, \quad \langle\eta\rangle=\begin{pmatrix}\delta v_{\eta_1}\\ v_\eta\end{pmatrix}, \quad \langle\chi\rangle=\begin{pmatrix}\delta v_{\chi_1}\\ \delta v_{\chi_2}\\ v_\chi+\delta v_{\chi_3}\end{pmatrix}, \quad \langle\xi\rangle=\begin{pmatrix}v_\xi+\delta v_{\xi_1}\\ \delta v_{\xi_2}\\ \delta v_{\xi_3}\end{pmatrix},\\
&\langle\rho\rangle=v_\rho, \qquad \langle\sigma\rangle=v_\sigma+\delta v_\sigma, \qquad \langle\vartheta\rangle=v_\vartheta,
\end{aligned}
\tag{B.11}
$$

$$
\langle\phi\rangle=\begin{pmatrix}v_\phi+\delta v_{\phi_1}\\ v_\phi+\delta v_{\phi_2}\\ v_\phi+\delta v_{\phi_3}\end{pmatrix}, \quad \langle\Delta\rangle=\begin{pmatrix}v_\Delta+\delta v_{\Delta_1}\\ v_\Delta+\delta v_{\Delta_2}\end{pmatrix}, \quad \langle\zeta\rangle=v_\zeta,
\tag{B.12}
$$

where the shifts δv_{η_2}, δv_ϑ, δv_ρ and δv_ζ have been absorbed into the redefinition of the undetermined v_η, v_ϑ, v_ρ and v_ζ respectively. With only terms linear in the shift δv retained and ignoring the $\delta v/\Lambda$ terms, the new minimization equations are still derived by the zeros of F-terms, i.e. the first derivative of new w_d in eq. (B.1) with respect to all driving fields. First the minimization equations for the set Φ^e are showed as follows

$$
\begin{aligned}
-2g_1v_\varphi\delta v_{\varphi_3}+g_2(v_\varphi\delta v_{\eta_1}+v_\eta\delta v_{\varphi_3})+\frac{v_\chi v_\phi^2}{\Lambda}A_1&=0\\
(4g_1v_\varphi+g_2v_\eta)\delta v_{\varphi_2}+\frac{v_\chi v_\phi^2}{\Lambda}A_2&=0\\
(-2g_1v_\varphi+g_2v_\eta)\delta v_{\varphi_1}+\frac{v_\chi v_\phi^2}{\Lambda}A_3&=0\\
g_3M_\chi\delta v_{\chi_1}+g_4(v_\eta\delta v_{\varphi_3}-v_\varphi\delta v_{\eta_1})+\frac{v_\chi v_\phi^2}{\Lambda}B_1&=0\\
g_3M_\chi\delta v_{\chi_3}+g_4v_\eta\delta v_{\varphi_2}+\frac{v_\chi v_\phi^2}{\Lambda}B_2&=0\\
g_3M_\chi\delta v_{\chi_2}+g_4v_\eta\delta v_{\varphi_1}+\frac{v_\chi v_\phi^2}{\Lambda}B_3&=0\\
2h_1v_\varphi\delta v_{\varphi_3}+2h_2v_\eta\delta v_{\eta_1}+\frac{v_\chi v_\phi v_\Delta}{\Lambda}C&=0
\end{aligned}
\tag{B.13}
$$

where the coefficients $A_{1,2,3}$, $B_{1,2,3}$ and C stand for the linear combinations of sub-leading coefficients

$$
\begin{aligned}
&A_1=3a_1-2a_3\frac{v_\Delta}{v_\phi}+a_5\frac{v_\Delta^2}{v_\phi^2}, && A_2=2a_3\frac{v_\Delta}{v_\phi}, && A_3=-3a_1-a_5\frac{v_\Delta^2}{v_\phi^2}\\
&B_1=3b_2-2b_4\frac{v_\Delta}{v_\phi}+b_6\frac{v_\zeta}{v_\phi}+b_8\frac{v_\Delta^2}{v_\phi^2}, && B_2=3b_1-2b_4\frac{v_\Delta}{v_\phi}+b_6\frac{v_\zeta}{v_\phi}+2b_7\frac{v_\Delta^2}{v_\phi^2}\\
&B_3=3b_2+4b_4\frac{v_\Delta}{v_\phi}+b_8\frac{v_\Delta^2}{v_\phi^2}, && C=0
\end{aligned}
\tag{B.14}
$$

The solutions for eq. (B.13) can be easily obtained as follows

$$
\begin{aligned}
&\delta v_{\varphi_1} = \frac{A_3}{4g_1}\frac{v_\chi}{\Lambda}\frac{v_\phi^2}{v_\varphi}, \qquad\qquad \delta v_{\varphi_2} = -\frac{A_2}{2g_1}\frac{v_\chi}{\Lambda}\frac{v_\phi^2}{v_\varphi},\\
&\delta v_{\varphi_3} = \frac{2g_1h_2A_1}{8g_1^2h_2 - g_2^2h_1}\frac{v_\chi}{\Lambda}\frac{v_\phi^2}{v_\varphi}\\
&\delta v_{\chi_1} = -\left[\frac{B_1}{g_3} + \frac{A_1g_4}{g_2^2h_1 - 8g_1^2h_2}\left(\frac{4g_1^2h_2}{g_2g_3} + \frac{g_2h_1}{g_3}\right)\right]\frac{v_\chi}{\Lambda}\frac{v_\phi^2}{M_\chi}\\
&\delta v_{\chi_2} = \left(\frac{g_4A_3}{2g_2g_3} - \frac{B_3}{g_3}\right)\frac{v_\chi}{\Lambda}\frac{v_\phi^2}{M_\chi}, \qquad \delta v_{\chi_3} = \left(-\frac{g_4A_2}{g_3g_2} - \frac{B_2}{g_3}\right)\frac{v_\chi}{\Lambda}\frac{v_\phi^2}{M_\chi},\\
&\delta v_{\eta_1} = \frac{2g_1h_1A_1}{g_2^2h_1 - 8g_1^2h_2}\frac{v_\chi}{\Lambda}\frac{v_\phi^2}{v_\eta}
\end{aligned}
\tag{B.15}
$$

The minimization equation for the shifted VEV δv_σ is obtained in the same way as follows

$$2q_1v_\sigma\delta v_\sigma + \frac{v_\varphi v_\chi v_\phi}{\Lambda}M = 0 \tag{B.16}$$

where the coefficients M is naively as

$$M = m_1 - m_2\frac{v_\eta}{v_\varphi} \tag{B.17}$$

Then eq. (B.16) admits the following solutions

$$\delta v_\sigma = -\frac{M}{2q_1}\frac{v_\phi}{\Lambda}\frac{v_\varphi v_\chi}{v_\sigma} \tag{B.18}$$

The equations for corrections $\delta v_{\xi_{1,2,3}}$ are simple as follows

$$
\begin{aligned}
4r_1v_\xi\delta v_{\xi_1} + r_2v_\rho\delta v_{\xi_1} &= 0\\
-2r_1v_\xi\delta v_{\xi_3} + r_2v_\rho\delta v_{\xi_3} + K\frac{v_\xi v_\chi v_\vartheta}{\Lambda} &= 0\\
-2r_1v_\xi\delta v_{\xi_2} + r_2v_\rho\delta v_{\xi_2} &= 0
\end{aligned}
\tag{B.19}
$$

where the coefficient K is easily solved as

$$K = k \tag{B.20}$$

and the solutions to the above minimization equations (B.19) are

$$\delta v_{\xi_1} = 0, \quad \delta v_{\xi_3} = \frac{K}{4r_1}\frac{v_\chi v_\vartheta}{\Lambda}, \quad \delta v_{\xi_2} = 0 \tag{B.21}$$

Despite of some shifts, $\delta v_{\xi_{1,2}}$, are exactly zero, most of the shifted VEVs δv_{Φ^e} are all of order in the interval $[\lambda_c^6\Lambda,\ \lambda_c^4\Lambda]$ from eqs. (B.15), (B.18) and (B.21), which imply the relative order of shifted VEVs with respect the LO VEVs of Φ^e are in the interval $[\lambda_c^4,\ \lambda_c^2]$, i.e.,

$\delta v_{\Phi^e}/v_{\Phi^e} \in [\lambda_c^4, \lambda_c^2]$. As illuminated in the end of section 3, the subleading corrections to $\langle\Phi^e\rangle$ should be smaller than the mass ratio $\frac{m_\mu}{m_\tau}$ or, more strictly $\frac{m_e}{m_\tau}$. The results above have shown the corrections are suitable for the model, and LO VEVs in eqs. (4.3) (4.6) and (4.9) are stable solutions even under NLO corrections.

At last for the Φ^ν sector we have the minimization equations

$$\begin{aligned}
f_1[v_\phi(\delta v_{\Delta_1} - \delta v_{\Delta_2}) + v_\Delta(\delta v_{\phi_2} - \delta v_{\phi_3})] + \frac{v_\phi^4}{\Lambda^2}C_1 &= 0 \\
f_1[v_\phi(\delta v_{\Delta_1} - \delta v_{\Delta_2}) + v_\Delta(\delta v_{\phi_1} - \delta v_{\phi_2})] + \frac{v_\phi^4}{\Lambda^2}C_1 &= 0 \\
f_1[v_\phi(\delta v_{\Delta_1} - \delta v_{\Delta_2}) + v_\Delta(\delta v_{\phi_3} - \delta v_{\phi_1})] + \frac{v_\phi^4}{\Lambda^2}C_1 &= 0 \\
2f_2 v_\phi(\delta v_{\phi_1} + \delta v_{\phi_2} + \delta v_{\phi_3}) + 2f_3 v_\Delta \delta v_{\Delta_1} + \frac{v_\phi^4}{\Lambda^2}C_2 &= 0 \\
2f_2 v_\phi(\delta v_{\phi_1} + \delta v_{\phi_2} + \delta v_{\phi_3}) + 2f_3 v_\Delta \delta v_{\Delta_2} + \frac{v_\phi^4}{\Lambda^2}C_2 &= 0 \\
2f_5 v_\phi(\delta v_{\phi_1} + \delta v_{\phi_2} + \delta v_{\phi_3}) + 2f_6 v_\Delta(\delta v_{\Delta_1} + \delta v_{\Delta_2}) + \frac{v_\phi^2 v_\zeta^2}{\Lambda^2}C_3 &= 0
\end{aligned} \tag{B.22}$$

where the coefficients C_1, C_2 and C_3 are

$$\begin{aligned}
C_1 &= 0 \\
C_2 &= 9(t_1+t_2) + 3(t_6+2t_7+t_8)\frac{v_\Delta^2}{v_\phi^2} + 3t_9\frac{v_\zeta^2}{v_\phi^2} + (2t_{10}+t_{11})\frac{v_\Delta^4}{v_\phi^4} + t_{12}\frac{v_\Delta^2 v_\zeta^2}{v_\phi^4} \\
C_3 &= 3u_1 + 2u_2\frac{v_\Delta^2}{v_\phi^2} + u_3\frac{v_\zeta^2}{v_\phi^2} + 9(u_4+2u_5)\frac{v_\phi^2}{v_\zeta^2} + 6(u_8+u_9)\frac{v_\Delta^2}{v_\zeta^2} + 2(2u_{10}+u_{11})\frac{v_\Delta^4}{v_\phi^2 v_\zeta^2}
\end{aligned} \tag{B.23}$$

The solutions to the eqs. (B.22) are also easily obtained as follows

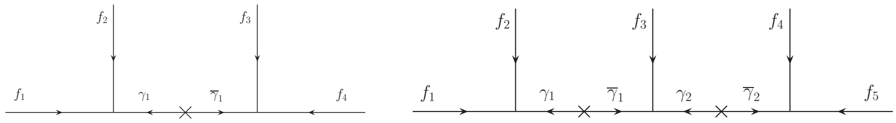
$$\begin{aligned}
\delta v_{\phi_1} = \delta v_{\phi_2} = \delta v_{\phi_3} &= \frac{2C_2 f_6 v_\phi^2 - C_3 f_3 v_\zeta^2}{(6f_3f_5 - 12f_2f_6)v_\phi}\frac{v_\phi^2}{\Lambda^2} \\
\delta v_{\Delta_1} = \delta v_{\Delta_2} &= -\frac{C_2 f_5 v_\phi^2 - C_3 f_2 v_\zeta^2}{(2f_3f_5 - 4f_2f_6)v_\Delta}\frac{v_\phi^2}{\Lambda^2}
\end{aligned} \tag{B.24}$$

Obviously all the shifts in three and/or two components of all scalar fields in Φ_1^e are different but within the same order of magnitude. All the shifts, however, in three and/or two components of all scalar fields in Φ^ν are exactly the same, means the small shifts are in the same direction of LO alignment. The result shows the stability of $\langle\Phi^\nu\rangle$, thus no soft terms are needed to drive the superpotential into desired minimum. The stable solutions of $\langle\Phi^\nu\rangle$ also guarantee the stability of TBM mixing in neutrino sector. Take into account the conditions in eq. (4.16), it is easy to check the relative order of shifted VEVs with respect to LO VEVs as following

$$\frac{\delta v_{\varphi_i}}{v_\varphi} \sim \frac{\delta v_{\eta_1}}{v_\eta} \sim \frac{\delta v_{\chi_i}}{v_\chi} \sim \lambda_c^4, \quad \frac{\delta v_\sigma}{v_\sigma} \sim \lambda_c^3, \quad \frac{\delta v_{\xi_3}}{v_\xi} \sim \lambda_c^2, \quad \frac{\delta v_{\xi_{1,2}}}{v_\xi} = 0, \quad \frac{\delta v_{\phi_i}}{v_\phi} \sim \frac{\delta v_{\Delta_i}}{v_\Delta} \sim \lambda_c^6 \tag{B.25}$$

Field	Γ_1	Γ_2	$\Gamma_3^{(r_i)}$	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}	Γ_{13}	Γ_{14}	Γ_{15}	Γ_{16}	Γ_{17}	Γ_{18}	Γ_{19}	Γ_{20}
SU(5)	$\bar{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{1}$	$\bar{\mathbf{5}}$	$\mathbf{5}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	$\mathbf{10}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{10}$	$\overline{\mathbf{10}}$	$\overline{\mathbf{15}}$	$\bar{\mathbf{5}}$	$\mathbf{1}$
S_4	3_1	1_2	r_i	1_1	1_2	1_2	1_1	1_2	1_2	3_2	1_1	1_1	1_1	1_1	1_2	3_2	1_2	1_2	3_2	3_2
Z_4	1	$-i$	1	-1	i	$-i$	1	$-i$	1	-1	i	-1	-1	1	-1	1	i	$-i$	$-i$	i
Z_6	ω^2	$-\omega$	1	-1	$-\omega^2$	$-\omega^2$	ω	-1	1	ω^2	ω	$-\omega$	ω^2	ω^2	ω^2	$-\omega$	$-\omega^2$	$-\omega$	ω^2	ω
Z_5	1	ω^4	1	ω	1	1	1	ω	ω^2	1	1	ω	ω^3	ω	ω	ω^2	1	1	1	ω
Z_2	-1	-1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	1	1	1	-1	-1	-1	1
$\mathrm{U}(1)_R$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 4. Transformation properties of the heavy messenger fields with direct masses in the model. The superscript r_i in $\Gamma_3^{(r_i)}$ stands for the representations 1_1, 2 and 3_1 of S_4.



(a) (b)

Figure 5. The supergraphs before integrating out the messenger fields of order 4 (left) and 5 (right) operators in the superpotential.

Field	f_1	f_2	f_3	f_4	γ_1
1	H_5	N^c	F	ϕ/Δ	Γ_1
2	H_{45}	T_3	T_2	σ	Γ_2
3	N^c	N^c	Φ^ν	Φ^ν	Γ_3
4	T_3	$H_{\bar{5}}$	F	φ	Γ_4

fields	f_1	f_2	f_3	f_4	f_5	γ_1	γ_2
1	T_2	H_5	T_2	σ	ϑ	Γ_5	Γ_6
2	H_5	T_3	T_3	Φ_i^ν	Φ_i^ν	Γ_7	$\Gamma_3^{1_1}$
3	T_2	$H_{\overline{45}}$	σ/ξ	F	χ/ζ	Γ_8	Γ_9/Γ_{10}
4	T_1	$H_{\overline{45}}$	σ	F	φ	Γ_{11}	Γ_{12}

Table 5. The operators corresponding to figure 5(a) (left) and figure 5(b)(right).

C The messenger sector

The higher dimensional operators of the effective superpotential can be obtained by integrating out the heavy messenger fields. The messenger fields Γ_i whose masses arise from singlets are listed in table 4. Here only half of the messenger fields are presented since each $\overline{\Gamma}_i$ takes opposite charges with respect Γ_i, which makes the direct mass term $M_{\Gamma_i}\Gamma_i\overline{\Gamma}_i$ is guaranteed by the symmetries. The messenger pairs Σ_i and $\overline{\Sigma}_i$ whose masses arise from the adjoint H_{24} (of the form $H_{24}\Sigma_i\overline{\Sigma}_i$) are listed in figure 7. Due to the amount of the operators and the variety of their possible contraction ways of S_4, not all of the operators are listed. Only those whose contributions to the entries of mass matrices non-vanishing at LO and small part of those being non-neglected at NLO (or NNLO) corrections are showed below. Note that Γ_3 has all the Z_N charges to be 1, thus we denote $\Gamma_3^{(r_i)}$ as one symbol Γ_3 for simplicity. In the left of table 5 Γ_3 contains all the possible S_4 representations, i.e., 1_1, 2 and 3_1, depend on the S_4 contraction ways of the fields in the corresponding operators. In the right table Γ_3 only include the invariant 1_1 of S_4. It is easy to realise the difference.

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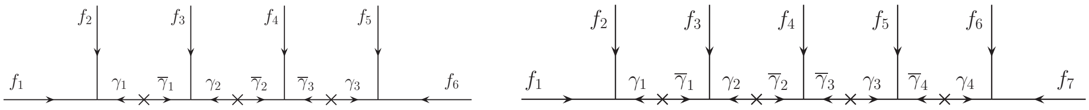


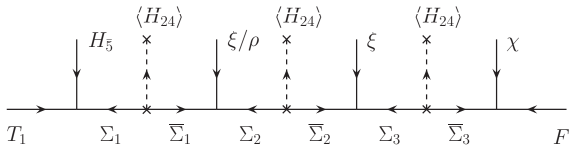
(a) (b)

Figure 6. The supergraphs of order 6 and 7 operators in the superpotential.

Field	f_1	f_2	f_3	f_4	f_5	f_6	γ_1	γ_2	γ_3
1	T_1	H_5	T_1	ζ	η	Δ	Γ_{13}	Γ_{14}	Γ_{15}
2	$H_{\overline{45}}$	T_2	σ	F	φ	η	Γ_8	Γ_9	Γ_{16}
3	H_{45}	T_2	T_3	σ	Φ_i^ν	Φ_i^ν	Γ_2	$\overline{\Gamma}_{10}$	$\Gamma_3^{1_1}$

Field	f_1	f_2	f_3	f_4	f_5	f_6	f_7	γ_1	γ_2	γ_3	γ_4
1	σ	T_2	H_{24}	F	$H_{\bar{5}}$	ϕ/Δ	ξ	Γ_{17}	Γ_{18}	Γ_{19}	Γ_{20}

Table 6. The operators corresponding to figure 6(a) (up) and figure 6(b) (down).



Field	Σ_1	$\overline{\Sigma}_1$	Σ_2	$\overline{\Sigma}_2$	Σ_3	$\overline{\Sigma}_3$
SU(5)	$\bar{\mathbf{5}}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{5}$
S_4	1_1	1_1	$3_1/1_1$	$3_1/1_1$	3_1	3_1
Z_4	1	-1	i	i	-1	1
Z_6	$-\omega^2$	1	$-\omega^2$	1	$-\omega^2$	1
Z_5	ω^4	ω	ω^3	ω^2	ω^2	ω^3
Z_2	1	1	-1	-1	1	1
$\mathrm{U}(1)_R$	0	0	0	0	0	0

Table 7. The supergraph of operators with messenger masses from the adjoint H_{24} and the assignments of GUT and flavor groups of corresponding messenger fields.

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